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Automated parametrical antenna modelling for ambient assisted living applications

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Abstract. In this paper a parametric modeling technique for a fast polynomial extraction of the physically relevant parameters of inductively coupled RFID/NFC (radio frequency identification/near field communication) antennas is presented. The polynomial model equations are obtained by means of a three-step procedure: first, full Partial Element Equivalent Circuit (PEEC) antenna models are determined by means of a number of parametric simulations within the input parameter range of a certain antenna class. Based on these models, the RLC antenna parameters are extracted in a subsequent model reduction step. Employing these parameters, polynomial equations describing the antenna parameter with respect to (w.r.t.) the overall antenna input parameter range are extracted by means of polynomial interpolation and approximation of the change of the polynomials' coefficients. The described approach is compared to the results of a reference PEEC solver with regard to accuracy and computation effort.

1 Introduction

For an energy-efficient and secure wireless medical data transmission in future Mobile Ambient Assisted Living (MAS) devices, novel antenna concepts in the near field region need to be investigated. This requires the examination of a wide variety of antenna systems during the design process to find the optimum configuration for a specific application. Thus, an adequate method that is able to accurately characterize the antennas on the one hand, and to allow for fast and efficient parameter sweeps on the other, is needed. While several approaches exist for the efficient determination of the coupling between spiral coils, the modeling of the actual antennas leads to more complex models, since phenomenon like eddy current losses need to be considered. This means a limiting factor for the applicability of accurate, but computationally expensive full wave 3-D solvers. Other approaches that can be used for a more efficient extraction of antenna parameters, such as analytical expressions or complexity reduction techniques, have the common benefit of reducing the computational effort to a certain degree compared to the full-wave solution, but show individual drawbacks: rules of thumb do not provide sufficient accuracy and analytical expressions exist only for a limited number of geometries usable for antenna modeling. By using Model Order Reduction (MOR) algorithms for the complexity reduction of RLC network models, such as PEEC models, the physical properties of the original model can usually not be preserved. In the reluctance-based method (Devgan et al., 2000) as well as the Fast Multipole Method (FMM) (Antonini, 2003) a simulation speed-up is being achieved by neglecting the weakly coupled elements. Due to the high aspect ratios as well as the close proximity of the current cells of inductively coupled coil models in the high frequency regime, the applicability of these two methods is limited (Scholz, 2010). In Scholz et al. (2010), an iterative procedure, described in more detail in Sect. 2, is presented where the modeling of the antenna impedances and the coupling between the antennas is separated. While this approach allows for fast spatial parameter sweeps as soon as the parameters for the equivalent antenna circuit models have been extracted, the necessity for the user to perform preceding reduction steps means a drawback.

Here, a parametric modeling technique for a fast polynomial approximation of the macro-model parameters of a single coil, depicted in Fig. 1, is being presented (see Sect. 3). The method is based on the PEEC method as well as the reduced self-impedance broadband models of Scholz et al. (2010). A major advantage is the wide range of validity

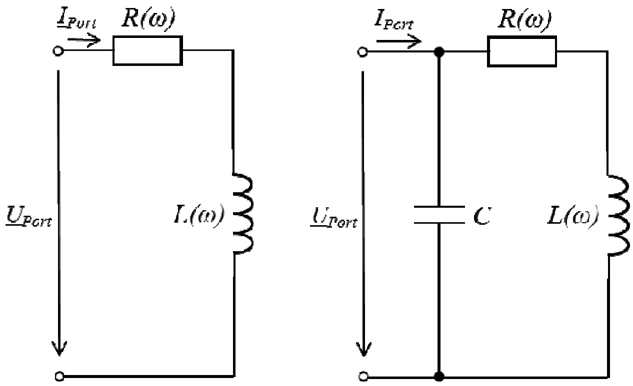

Fig. 1. MQS (left) and QS (right) equivalent circuit model of a coil.

of the established model equations, allowing for the determination of antenna RLC parameters of an entire antenna class, thereby freeing the designer from time consuming model reduction steps. In a case study, the DC antenna inductance for the class of rectangular coils with rectangular cross section is modeled and compared to the results of a reference PEEC solver concerning accuracy and simulation time (see Sect. 4).

2 PEEC and reduced broadband self-impedance models

As the parametric modeling technique presented here is based on PEEC as well as reduced broadband self-impedance models, the two methods are outlined in short in the following.

2.1 PEEC models

The PEEC method is an integral equation based numerical method for the electromagnetic characterization of arbitrary three-dimensional interconnection structures, developed by Ruehli (1974). By means of the Mixed Potential Integral Equations (MPIE), the electric field is expressed as a function of the electric potential $\phi(\underline{r},t)$ and vector potential $\underline{A}(\underline{r},t)$:

$$\underline{E} = -\mathrm{grad}\varphi - j\omega\underline{A}. \quad (1)$$

The electric field in Eq. (1) is substituted with ohm's law $\underline{J}/\kappa$ and the potentials $\phi(\underline{r},t)$ and $\underline{A}(\underline{r},t)$ are substituted with the Green's functions containing the source terms ρ and $\underline{J}$. The interconnection structure is then partitioned into inductive/resistive volume cells containing the currents and capacitive surface cells containing the surface charges. After developing the source terms by constant basis functions and applying the Galerkin method, the terms in Eq. (1) can be interpreted as partial resistances, capacitances and inductances, where the capacitive and inductive mutual couplings are being considered. Thus, a transition to the network domain is obtained where the partial elements form an equivalent circuit that can be analyzed by circuit solvers, e.g. SPICE. A system of equations can then be set up, e.g. by means of the Modified Nodal Analysis (MNA) or a mesh-based approach, where the system matrices are set up by means of the partial elements and the source terms form the states of the system. The found solutions for the source terms may then be used to determine the EM fields of the considered problem.

As the inductively coupled coil antennas considered here are designed for the 13.56 MHz frequency range, retardation effects can be neglected, making it sufficient to use the non-retarded PEEC method based on the quasi static assumption of Maxwell's equations.

2.2 Reduced broadband self-impedance models

In Scholz et al. (2010), an antenna parameter extraction technique is presented where at first the antenna impedance of each individual antenna is determined in an iterative procedure, followed by the calculation of the mutual inductive coupling between the antennas. Finally, the extracted parameters are assembled to a *reduced broadband self-impedance model*.

In the first step, a magneto quasi static (MQS) and a quasi static (QS) high complexity antenna model is extracted by means of a number of PEEC simulations, whereas in the first case the capacitive elements are being neglected (see Fig. 1, left) and incorporated in the second case (see Fig. 1, right). By means of the obtained magneto quasi static and quasi static antenna impedances Z_{MQS} and Z_{QS}, the lumped inductance L and resistance R, as well as the parasitic capacitance C of the coil (see Fig. 1) can be extracted from the PEEC models, using network based extraction-techniques (Scholz et al., 2010). In the second step, the mutual coupling M between the antennas is efficiently calculated by means of a filamentary approximation of the coil's segments (Grover, 2004) as well as a modified version of the well-known Greenhouse method (Greenhouse, 1974) adapted for the PEEC method (Reinhold et al., 2007).

Based on this procedure, the required input parameters for broadband models valid from DC to the first resonance can be obtained. The frequency dependent behavior of the impedance is then approximated by means of a ladder model obtained from e.g. a continued fraction expansion of the expression describing $Z(\omega)$ (see Engin et al., 2005; Scholz et al., 2010).

While the calculation of the mutual inductance M can be performed very efficient with the aid of the before mentioned measures, the extraction of the RLC parameters for the actual antennas demands several costly PEEC simulations. In Sect. 3, an approach for the automated extraction of polynomials allowing for the fast approximation of the RLC coil parameters is presented that releases the designer from having to perform preceding reduction steps.

Table 1. Input parameters and range.

Parameter	Range
Outer length l_x	20–100 mm
Outer length l_y	20–100 mm
Trace width w	1.0–2.6 mm
Track spacing s	1.0–2.6 mm
Trace height h	35 µm
Number of turns N_t	1–3

3 Automated parametrical modelling of rectangular spiral inductors

The automated parametrical antenna modelling technique presented in this contribution is illustrated by means of the approximation of the coil's DC inductance L_{DC} in the following, but can be applied for the approximation of any other RLC antenna parameter as well. This is due to the fact that the procedure for the extraction of the polynomial equations used for the approximation of a certain antenna parameter is the same for any parameter, making it a fully automated approach. A main advantage is the wide range of validity of the polynomial equations, making it possible to approximate a certain antenna parameter for any input parameter combination with one set of polynomial equations only. Thus, by means of the extracted equations, the antenna parameters for a whole class of antennas are being calculated. Here, the class of rectangular coils with rectangular cross section is focused on.

As mentioned earlier, the presented technique is based on the PEEC method as well as the *reduced broadband self-impedance models*. The input parameters needed for the extraction of the polynomial equations are obtained by a number of PEEC simulations performed within the antennas' geometrical input parameter range (see Table 1), followed by the parameter extraction methodology of the reduced broadband self-impedance models outlined in Sect. 2.2. The extracted parameters, in this example the DC inductance L_{DC}, serve as the input values for the program source code used for the extraction of the polynomial equations.

To be able to determine the DC inductance L_{DC} of the coil over the whole geometrical input parameter range, it is necessary to define a minimum step size Δ_{Step} for each parameter of Table 1. The step size needs to be chosen so that the change of the sought antenna parameter L_{DC} w.r.t. a respective input parameter smaller than the chosen step size does not exceed a value of interest for the user. Here it is chosen $\Delta_{Step,l} = 1$ mm for the outer lengths l_x, l_y and $\Delta_{Step,w} = \Delta_{Step,s} = 0.1$ mm for the track width w and the track spacing s. Thus, the input parameter range is subdivided into $N_{l_x} = N_{l_y} = 81$ steps for l_x and l_y and $N_w = N_s = 17$ steps for w and s, leading to a number of input parameter combinations of

$$N_{Komb} = N_{l_y Steps} \cdot N_{l_x Steps} \cdot N_{w Steps} \cdot N_{s Steps} \approx 1.89 \times 10^6. \quad (2)$$

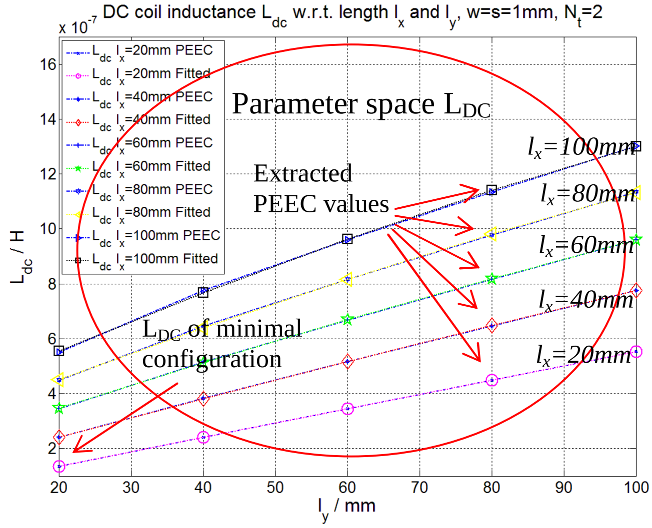

Fig. 2. Parameter space of DC inductance L_{DC} w.r.t. input parameters l_x, l_y.

In Fig. 2, the parameter space of the DC inductance $L_{DC}(l_x, l_y)$ w.r.t. two of geometrical input parameters is shown, namely the two outer lengths l_x and l_y.

The array of curves describes the DC inductance w.r.t. the outer length l_y, and every curve of the array represents a discrete value of the outer length l_x (see legend of Fig. 2). The dots mark the extracted inductance values from the PEEC simulations. The curves are approximated by polynomial interpolation using polynomials of order two. While Fig. 2 illustrates the modelling of the DC inductance w.r.t. only two input parameters, the main challenge is to model the inductance w.r.t. all five geometrical input parameters listed in Table 1 (trace height not counted). Obviously, every geometrical input parameter added to the modelling procedure needs two be incorporated w.r.t. all input parameters that were regarded in the preceding modelling steps. Starting point of the modelling procedure is to choose a minimal antenna configuration, describing the smallest antenna that can be modelled. The values of the minimal configuration correspond to the smallest values of the parameter range in Table 1 for each input parameter (see also Fig. 2).

In the first modelling step, the DC inductance is modelled w.r.t. the outer length l_y. For all other geometrical input parameters, the values of the minimal antenna configuration are chosen. Using polynomial interpolation, a polynomial $L_{DC}(l_y)$ is obtained:

$$L_{DC}(l_y) = \sum_{n=0}^{2} K_{n,l_y} \cdot l_y^n \quad (3)$$

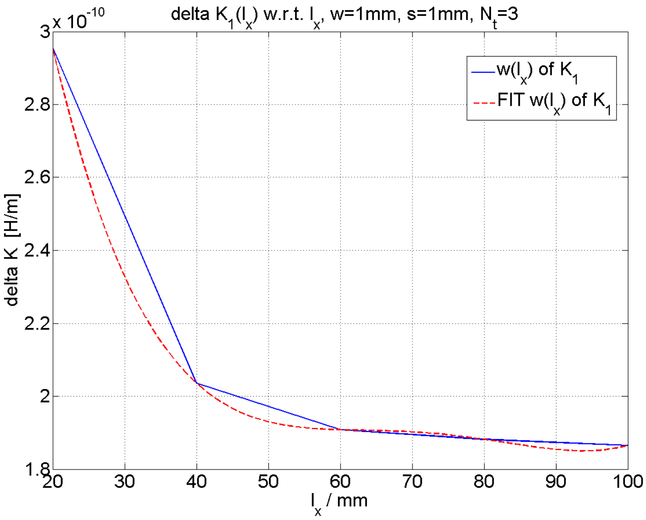

Fig. 3. Interpolation (dashed) of the change of coefficient $\Delta K_1(l_x)$ (continuous).

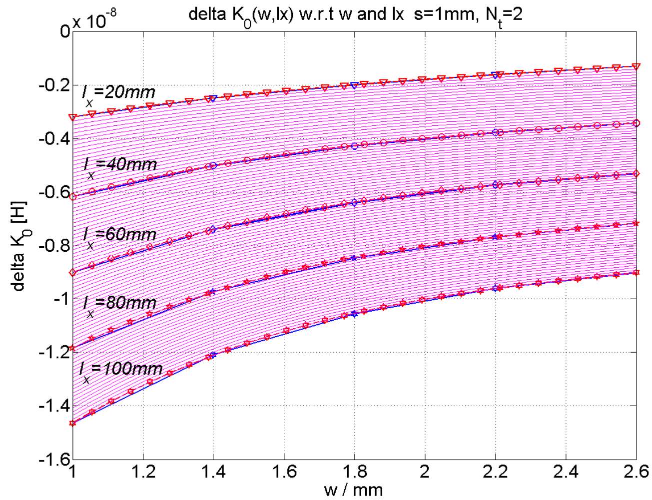

Fig. 4. Interpolation (dashed marked) of the change of coefficient $\Delta K_0(w,l_x)$ (marked and unmarked continuous).

Equation (3) describes the bottommost curve in Fig. 2. Next, the second length parameter, l_x, is accounted for by investigating the change of the coefficients $K_{2,l_y} - K_{0,l_y}$ in Eq. (3) w.r.t. l_x. First, the rest of the curves of the array in Fig. 2 are modeled by means of polynomial interpolation accordingly to the extraction of Eq. (3). Then, the PEEC simulations and L_{DC} parameter extraction is performed again for l_x, this time with a displacement of l_x of the step size $\Delta_{\mathrm{Step},l} = 1$ mm compared to the original simulations. Finally, the change of the coefficients $K_{2,l_y} - K_{0,l_y}$ is determined by comparison of the original coefficients with the coefficients obtained for the displaced curves at five discrete equidistant points over the input range of the parameter l_x.

In Fig. 3, the piece-wise linear curve represents the determined change of coefficient $\Delta K_{1,m}(l_x)$, where the indices m describes the subdivision of the range of $\Delta K_1(l_x)$ into $N_{l_x\mathrm{Steps}}$ (see Eq. 2) discrete values. This curve is fitted by means of a polynomial of third order, illustrated by the dashed curve. The change of the original coefficients $K_{2,l_y} - K_{0,l_y}$ w.r.t. l_x is expressed by means of a new set of coefficients $K_{2,l_x} - K_{0,l_x}$, see Eq. (4). For a certain l_x, these coefficients are obtained by summing the first M coefficient changes $\Delta K_{1,m}(l_x)$, where M is defined according to Eq. (4) and $l_{x,\mathrm{Min}}$ describes the lower bound of the input parameter range of l_x.

$$K_{n,l_x} = \sum_{m=1}^{M} \Delta K_{n,m}(l_x) \text{ with } n=0,1,2 \text{ and } M = \frac{l_x - l_{x,\mathrm{Min}}}{\Delta_{\mathrm{step},l}} \quad (4)$$

To obtain the DC inductance $L_{\mathrm{DC}}(l_x,l_y)$ w.r.t. l_y and l_x, the new coefficients $K_{2,l_x} - K_{0,l_x}$ are added to the respective coefficients $K_{2,l_y} - K_{0,l_y}$ according to Eq. (5):

$$L_{\mathrm{DC}}(l_y,l_x) = \sum_{n=0}^{2} \left(K_{n,l_y} + K_{n,l_x}\right) \cdot l_y^n \quad (5)$$

To suggest for the influence of the track width w, this parameter needs to be incorporated in the modeling process dependent on the previously considered input parameter l_x. Other than this, a similar procedure as for the incorporation of the parameter l_x is pursued. The change of the coefficients $K_{2,l_y} - K_{0,l_y}$ and $K_{2,l_x} - K_{0,l_x}$ in Eq. (4) w.r.t. w and l_x is determined by repeating the original PEEC simulations with w displaced by the step size $\Delta_{\mathrm{Step},w} = 0.1$ mm and calculation of the change of the coefficients by comparing the displaced with the original results. In Fig. 4, the continuous marked curves represent the change of coefficient $\Delta K_{0,m}(w,l_x)$, where each individual curve describes the change of coefficient w.r.t. the track width w for a certain discrete value of l_x over the input range of l_x.

The dashed marked curves describe the polynomial approximation of the original curves by means of third order polynomials. An array of thin lined curves between two dashed marked curves illustrates the approximation of the value range between two adjacent dashed marked curves. As two adjacent dashed marked curves correspond to a length difference of $\Delta l_x = 20$ mm in this example, the value range is subdivided by means of the step size $\Delta_{\mathrm{Step},l} = 1$ mm, resulting in 19 thin lined curves between every pair of dashed marked curves. This is achieved by a linear subdivision of the value range between two respective polynomial coefficients of the same degree, belonging to two adjacent dashed marked curves. Following the procedure of the incorporation of the input parameter l_x, a new set of coefficients $K_{2,w} - K_{0,w}$ is extracted for a certain track width to be modeled. $K_{2,w} - K_{0,w}$ are again obtained by summation of the first M coefficient changes $\Delta K_{2,m}(w,l_x) - \Delta K_{0,m}(w,l_x)$, see Eq. (6).

$$K_{n,w} = \sum_{m=1}^{M} \Delta K_{n,m}(w,l_x) \text{ with } n=0,1,2 \text{ and } M = \frac{w - w_{\mathrm{Min}}}{\Delta\mathrm{step}} \quad (6)$$

To obtain the DC inductance $L_{\mathrm{DC}}(w, l_x, l_y)$ w.r.t. w, l_y and l_x, the new coefficients $K_{2,w} - K_{0,w}$ are added to the respective coefficients $K_{2,l_y} - K_{0,l_y}$ and $K_{2,l_x} - K_{0,l_x}$ according to Eq. (7):

$$L_{\mathrm{DC}}(l_y, l_x, w) = \sum_{n=0}^{2} \left(K_{n,l_y} + K_{n,l_x} + K_{n,w} \right) \cdot l_y^n \quad (7)$$

The inclusion of the track space s in the modeling of the DC inductance $L_{\mathrm{DC}}(s, w, l_x, l_y)$ follows a similar procedure as described for the input parameters l_x and w. Again, the incorporation of s needs to be accomplished w.r.t. the input parameters that were regarded for in the preceding modeling steps, namely l_x and w. This is realized in two steps by means of the extraction of the coefficient change $\Delta K_{2,m}(s, w, l_x) - \Delta K_{0,m}(s, w, l_x)$ through comparison of the original with displaced PEEC simulations (displacement $\Delta_{\mathrm{Step},s} = 0.1\,\mathrm{mm}$):

first, a coarse subdivision of the parameter range of l_x and w in e.g. 5 subsections is carried out. Then, for every subdivision point of w the changes of coefficients $\Delta K_{2,m}(s, w, l_x) - \Delta K_{0,m}(s, w, l_x)$ are determined at every subdivision point of l_x. Next, after approximating the three functions $\Delta K_{2,m}(s, w, l_x) - \Delta K_{0,m}(s, w, l_x)$ by means of third order polynomials, for every adjacent pair of coefficients of the same degree of these polynomials, a linear subdivision is undertaken corresponding to the subdivision of the range between two subdivision points of l_x. By this, an approximation of the change of coefficients $\Delta K_{2,m}(s, w, l_x) - \Delta K_{0,m}(s, w, l_x)$ over the whole value range of the input parameter l_x is obtained at each of the five discrete subdivision points of the input parameter w.

In the second step, approximations over the whole input range of w, i.e. between the subdivision points of w, need to be found. To that end, the family of polynomials describing the change of coefficients $\Delta K_{2,m}(s, w, l_x) - \Delta K_{0,m}(s, w, l_x)$ over the whole value range of l_x at one subdivision point w is compared to the corresponding family of polynomials of the adjacent subdivision point w. This is carried out for all pairs of subdivision points w, again by means of linear subdivision of adjacent pairs of coefficients of the same degree.

By means of this two-step procedure, a new set of coefficients $K_{2,s} - K_{0,s}$ in Eq. (8) is obtained from the summation of the change of coefficients $\Delta K_{2,m}(s, w, l_x) - \Delta K_{0,m}(s, w, l_x)$.

$$K_{n,s} = \sum_{m=1}^{M} \Delta K_{n,m}(s, w, l_x) \text{ with } n = 0, 1, 2 \text{ and } M = \frac{s - s_{\mathrm{Min}}}{\Delta \mathrm{step}} \quad (8)$$

To determine the DC inductance $L_{\mathrm{DC}}(s, w, l_x, l_y)$, the a new set of coefficients $K_{2,s} - K_{0,s}$ is added to the coefficients extracted in the preceding steps.

$$L_{\mathrm{DC}}(l_y, l_x, w) = \sum_{n=0}^{2} \left(K_{n,l_y} + K_{n,l_x} + K_{n,w} + K_{n,s} \right) \cdot l_y^n \quad (9)$$

Table 2. Comparison of simulation time.

Method	Turns N_{t}	Simulation Time
Parametric modeling	arbitrary	0.25 s
PEEC	1	7.52 s
PEEC	3	67.04 s
PEEC	5	183.40 s

The remaining input parameter to be included in the modeling process is the number of turns N_{t}. Contrary to the input parameters considered so far, this parameter attains only integers. No intermediate values have to be considered. Thus, it seems natural to incorporate this parameter differently. While it might be possible to incorporate the last input parameter N_{t} in the same manner as the other parameters, it was decided here for reasons of simplification to repeat the modeling procedure described in the preceding N_{t} number of times. This means that N_{t} equations of type (9) describe a class of antennas to be modeled.

The polynomial equation extraction procedure outlined here was illustrated using the example of the DC inductance L_{DC}, but is applicable for any other antenna parameter without modification of the program code. Thus, the input parameter extraction for the broadband ladder models described in Sect. 2.2 is straightforward.

4 Validation of the method

In the following, the validation of the presented modelling approach is made by means of comparison with the PEEC method and the *reduced self-impedance broadband models*.

Whereas in the latter, the designer needs to perform a reduction of the full PEEC models to obtain the reduced model for every antenna configuration, this is not necessary for the automated parametrical modelling technique: Here, the designer obtains a set of polynomial equations valid for a class of antennas to extract the physically relevant parameters. All time consuming reduction steps were carried out in the course of the geometrical input parameter determination needed for the equation extraction, as described at the beginning of Sect. 3. The actual equation extraction is fully automated for all antenna parameters and demands only a few seconds. Regarding the computational effort of the parameter extraction with the new method, the simulation time for three different spiral coils is shown in Table 2 in comparison with full PEEC simulations. For all three coils holds: $w = s = 2\,\mathrm{mm}$, $l_x = l_y = 50\,\mathrm{mm}$ and $h = 35\,\mu\mathrm{m}$. The PEEC simulations were performed by means of a simulator designed in the course of this work. All simulations were performed on an Intel Core i3 Processor with 4 GByte RAM. The simulations were carried out at a single frequency point

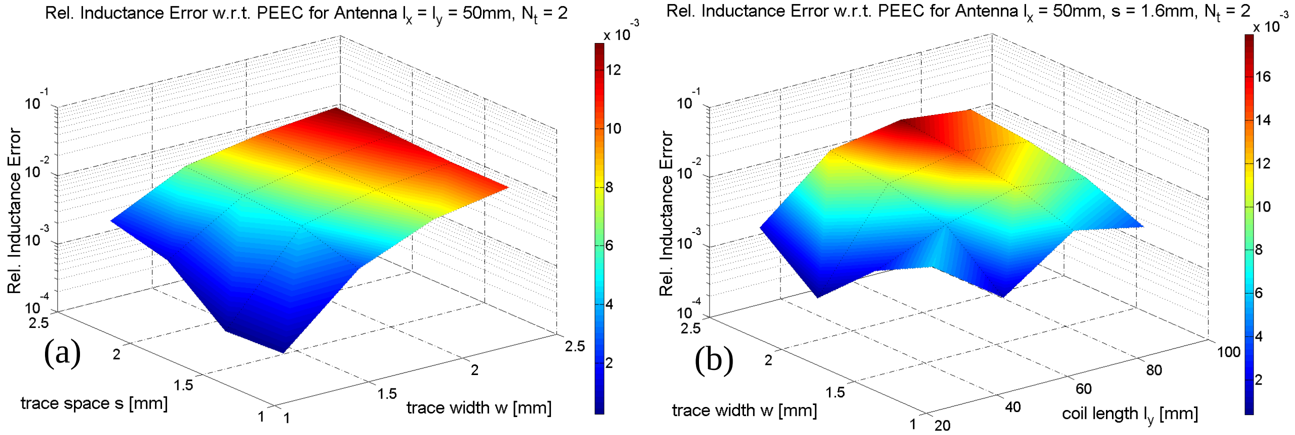

Fig. 5. Relative inductance error w.r.t. trace space s and trace width w **(a)** and w.r.t. trace width w and length l_y **(b)**.

at $f_{DC} = 1$ Hz. While the simulation time raises exponentially with the number of turns for the PEEC simulations, it remains constant for the parametrical modeling technique, as it does not depend on the configuration of the antenna. The total simulation time is less than a second for the new approach, while the PEEC simulation time exceeds one minute for a three turn antenna (Table 2). Concerning the accuracy of the new approach, 256 simulations were performed with the PEEC reference solver over the input parameter range. The geometrical input parameters were chosen as far away as possible from the input parameters of the original PEEC simulations, on which the equation extraction was based on. It is assumed that the modeling error is largest for these values. Figure 5a shows the relative inductance error w.r.t. the trace width w and the trace space s for a two-turn antenna with the outer lengths $l_x = l_y = 50$ mm. The rel. error lies below ~ 1.3 % for all values. It rises with increasing trace width and space and is more sensitive to the trace width than to the trace space. Figure 5b shows the relative inductance error w.r.t. the trace width w and the outer length l_x for a two-turn antenna with $l_x = 50$ mm and $s = 1.6$ mm. Here, the rel. error lies below ~ 1.7 % for all values. The maximum relative error w.r.t. all 256 reference values is ~ 2.2 % and the mean relative error over all values is ~ 0.6 %. Thus, a very good agreement between the extracted parameters of the parametrical modeling technique and the PEEC reference values is found.

5 Conclusions

In this contribution a parametrical modelling approach for the fast characterization of inductively coupled coils was presented. Based on the PEEC method and the reduced broadband self-impedance models, polynomial equations for the determination of the coil's DC inductance were extracted. The approach is fully automated in the sense that it is applicable for the extraction of all physically relevant RLC coil parameters without any modification. Further, its usage is not limited to the PEEC method, but can be applied universally to any numerical method that is able to characterize antennas by means of lumped element models. Numerical results show a good agreement with the PEEC method within the input parameter range. The mean relative error is ~ 0.6 % and the maximum relative error is ~ 2.2 % for a parameter sweep of 256 antenna configurations. With a computation time of below one second for the parameter extraction, the presented technique is very efficient compared to full-wave solvers. In contrast to other model reduction methods, the user is freed from having to perform reduction steps to obtain a reduced model. Another advantage is the easy handling for the user, as the geometrical input parameters simply need to be entered and no antenna geometries or script files need to be established. The extension to broadband models by means of physically motivated ladder-models is straightforward.

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