

# Examination of the demodulation effect of two-tone disturbances on nonlinear elements

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**Abstract.** The effect of two disturbing sources with close frequencies on a nonlinear circuit is investigated. It is shown that low-frequency oscillations occur in such a circuit. These low-frequency oscillations represent a significant threat to the electronic equipment, since standard filters do not suppress low-frequency harmonics. On the basis of Picard's iterations the synthesis of a protection system against low-frequency oscillations is performed.

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## 1 Introduction

Nowadays, there is a marked trend for the increase in frequency and shifting of the amplitude maximum of noise fields into the high-frequency region (Keiser, 1979; Paul, 1992; Ianoz et al., 1997). The shielding factor of non-ideal shields is reduced with increasing frequency (for example, the increase in the transfer impedance of cables with braided shields (Kaden, 1959; Lee and Baum, 1975; Demoulin, 1981; Gonschorek and Tiedemann, 2000; Demirchjan et al., 2003). Therefore, problems of protecting electronic devices, as a whole, become complicated with increasing frequency, and this trend will prevail in future.

In the present paper the mechanism for the occurrence of low-frequency noise due to the demodulation of the external two-tone high-frequency electromagnetic field is examined. The two-tone high-frequency electromagnetic field penetrates into the internal (protected) volume of the electronic device and generates a low-frequency noise that propagates further into electronic circuits. This paper also demonstrates the universal way of suppressing this effect.

The low-frequency disturbances can produce a number of problems for the electronic equipment, among which we will note the following:

- The frequencies in the  $(1-10^3)$  kHz range are considered here as low-frequency noise, and can be close to operational frequencies of analog electronic devices. In this case, the low-frequency noise directly distorts a useful signal of the electronic device.
- The low-frequency noise can shift the operating points of transistor cascades of electronic circuits substantially and thereby changing their operating conditions.
- The low-frequency noise can be a source of dynamic instabilities in complex electronic systems.

It should be noted also that the low-frequency noise has weak attenuation in the circuits of the electronic equipment. Usually, internal filters for the suppression of low-frequency noise are not provided. Therefore, a low-frequency noise, if it occurs in an electronic system, can become “a source of problems” in those units which are located “electrically far” from the place of its origin.

The two-tone high-frequency electromagnetic field can be originated by different mechanisms. One of these is shown in Fig. 1. The technical system located in a shielded volume is excited by an external electromagnetic field. Due to selective frequency properties of the cavity (Tkachenko and Vodopianov, 1998; Krauthaeuser et al., 2002; Tkachenko et al., 1999; Krauthaeuser et al., 2002) a two-tone signal may be produced.

Another possibility of obtaining two-tone high-frequency signals is the excitation of a system containing cables of different length which also have selective frequency properties. If the lengths of the cable shields are close to each other (but not equal) there may occur two-tone signals on the interior conductors of the cable due to the action of the cable transfer-impedance. One also can imagine a two-tone generator outside the system which very effectively may disturb it.

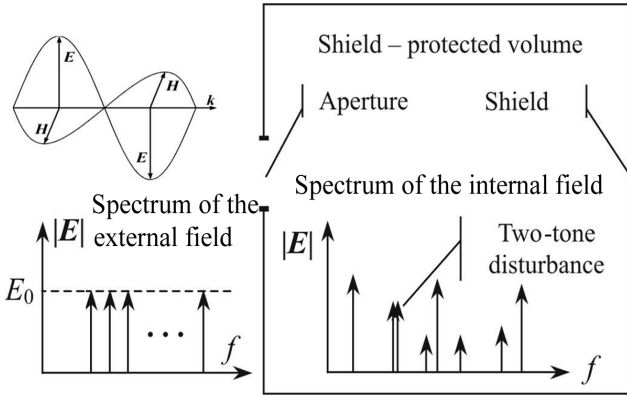


Fig. 1. Two-tone signal due to the excitation of a cavity.

## 2 The action of two-tone high-frequency excitation on a nonlinear system

We examine the origin of a low-frequency disturbance which might become a typical EMC problem. The considered system (Fig. 2) contains: the load  $R=50\text{ k}\Omega$  which models the input resistance of some electronic device (1 in Fig. 2), the source of the two-tone high-frequency disturbance:

$e_1(t) = 600 [\cos(\omega_1 t) + \cos(\omega_2 t)]\text{ V}$ ,  $\omega_1 = 15 \cdot 2\pi \cdot 10^6\text{ s}^{-1}$ ,  $\omega_2 = 16 \cdot 2\pi \cdot 10^6\text{ s}^{-1}$  (2 in Fig. 2), the simplest ( $L=10\text{ mH}$ ) low-pass filter (3 in Fig. 2), and a nonlinear load (4 in Fig. 2)  $u_{nl} = 2768 \cdot i - 4932 \cdot i^2 + 3033 \cdot i^3$  where  $i$  is measured in mA. In order to obtain the correct unit for the voltage  $u$  (nl), the necessary units have to be thought to be associated with the numerical numbers.

To calculate the steady state solution of the electrical circuit of Fig. 2 we apply Picard's iterations (Leon and Shaefer, 1977; Danilov, 1987; Rugh, 1981). After three iterations we get  $u_R(t)$ , including harmonics of the frequencies  $(\omega_2 - \omega_1)$ ,  $\omega_1$ , and  $\omega_2$  (the rest of the harmonics are small, so they are not listed), as follows:

$$u_R(t) = [6.094 + 6.49 \sin((\omega_2 - \omega_1)t) + 14.5 \sin(\omega_1 t + 0.962) + 14.0 \sin(\omega_2 t + 0.978)]\text{ V}. \quad (1)$$

The amplitude spectrum of  $u_R(t)$  is shown in Fig. 2. As is obvious from Fig. 2 substantial low-frequency components are formed due to the nonlinear  $RL$ -circuit. It turns out that the amplitudes of the low-frequency oscillations are approximately two times less than those of the high-frequency oscillations. The resistor  $R$  and the nonlinear element model certain appropriate resistances of the electronic circuit. In practice a large number of different real electronic circuits may be connected to the nodes  $\alpha - \alpha'$  and  $\beta - \beta'$ . The low-frequency oscillations resulting from the examined demodulation phenomenon may propagate along these terminated circuits, encountering no barriers. This is due to the fact that, usually, internal low-frequency filters are not installed. Therefore, as mentioned in the Introduction, the low frequency oscillations represent a special threat to electronic equipment.

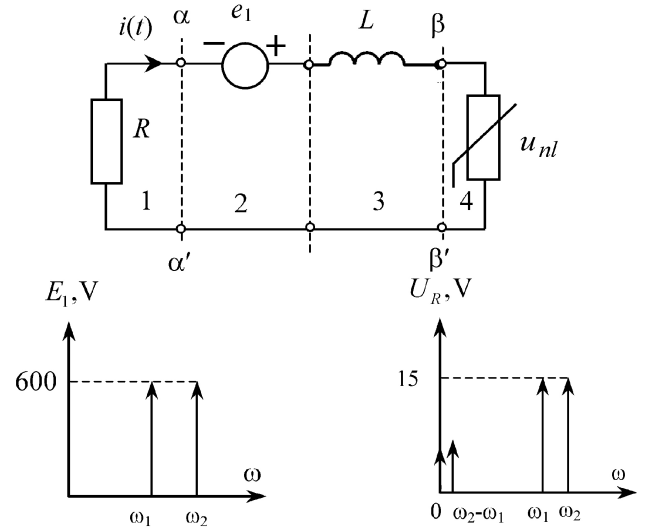


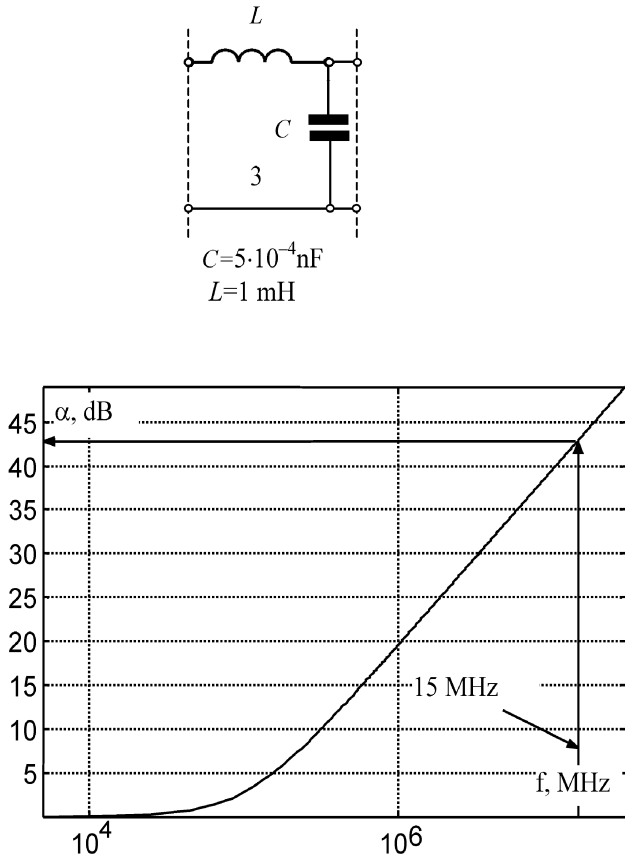
Fig. 2. The action of two-tone high-frequency excitation on a nonlinear system.

The more detailed consideration of the filter (unit 3 in Fig. 2) does not alter (neither qualitatively nor quantitatively) the nature of the demodulation phenomenon. To show this we replace the filter in the circuit of Fig. 2 by the filter which is presented in Fig. 3. New circuit parameters are:  $e(t) = 1500 [\cos(\omega_1 t) + \cos(\omega_2 t)]\text{ V}$ , the resistor  $R_s = 50\text{ k}\Omega$  is connected in series with the nonlinear load like before. The frequency dependence of the damping factor  $\alpha$  of the  $LC$ -low-pass filter is also presented in Fig. 3. For this calculation we assume the filter to be loaded with the linear term of the nonlinear resistor.

The plot of the voltage of the resistor  $R_s = 50\text{ k}\Omega$  (for one period) and the corresponding amplitude spectrum are shown in Fig. 4. We observe that substantial low-frequency components are also formed in the nonlinear circuit which contains the  $LC$ -filter. The amplitude of the low-frequency oscillation is approximately two times less than that of the high-frequency oscillations.

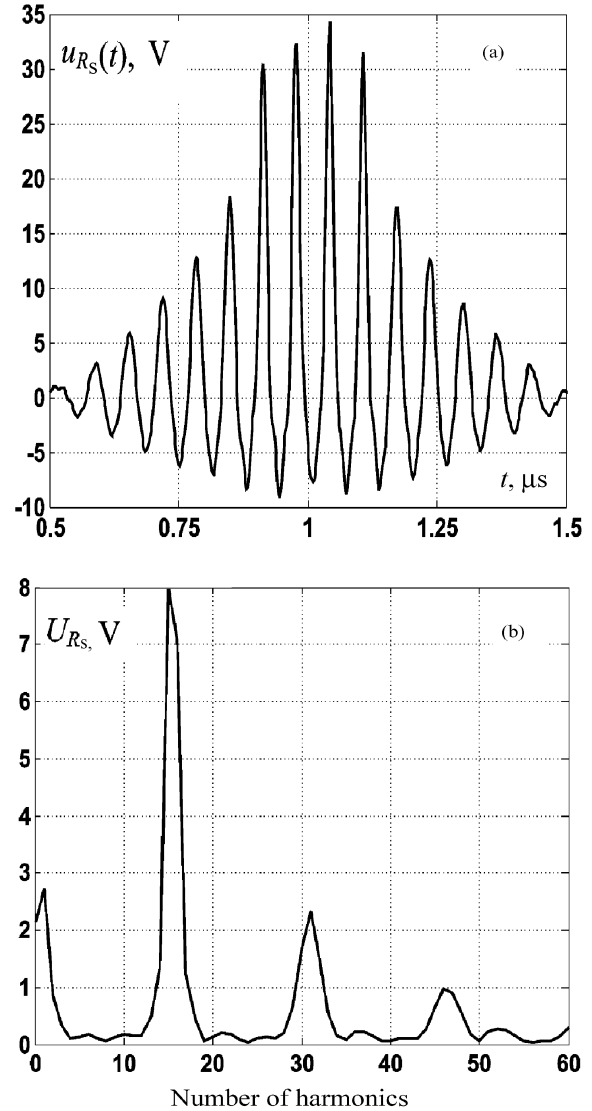
We draw some interim conclusions:

- Under the action of a two-tone excitation with frequencies  $\omega_1$  and  $\omega_2$  on a nonlinear system low-frequency oscillations with frequency  $\Delta\omega = \omega_2 - \omega_1$  are generated. Various mechanisms to generate two-tone oscillations are considered in the Introduction and can give rise to rather close frequencies  $\omega_1$  and  $\omega_2$ . Therefore, the low-frequency noise appearing under such excitation can have a frequency which is several (three and more) orders of magnitude less than  $(\omega_1 + \omega_2)/2$ . Thus, the examined mechanisms can be considered as low-frequency excitations of protected volumes by a high-frequency field and also as mechanisms which shift energy from a high-frequency field into a low-frequency one.



**Fig. 3.** Frequency dependence of the damping factor  $\alpha$  of the LC-low-pass filter.

- The low-frequency noise occurring in certain nonlinear circuits can further propagate along electric circuits of the protected device, because special means which limit this propagation are not provided, and its own attenuation is weak (due to the low frequency). Thus, it is expedient and necessary to provide protective means against low-frequency noise.
- At the presence of nonlinear elements in a protected device (that always can be assumed for electronic devices) and an assumed two-tone excitation, several repeatedly reflected noises are rather probable. Then there will, however, arise a problem, because the frequency of that low-frequency noise is actually unknown at the beginning of an EMC investigation. This is due to many aspects. It strongly depends on the individual features of the specific device, such as different cable-layouts, relative positions of apertures in shielded volumina, etc.
- Standard means to suppress low-frequency noise propagation, for example filters, will probably be inapplicable in the examined situation, since the noise frequency is not determined. The installation of filters with excessively wide stop bands can create problems for the transmission of useful signals. For mobile objects it is



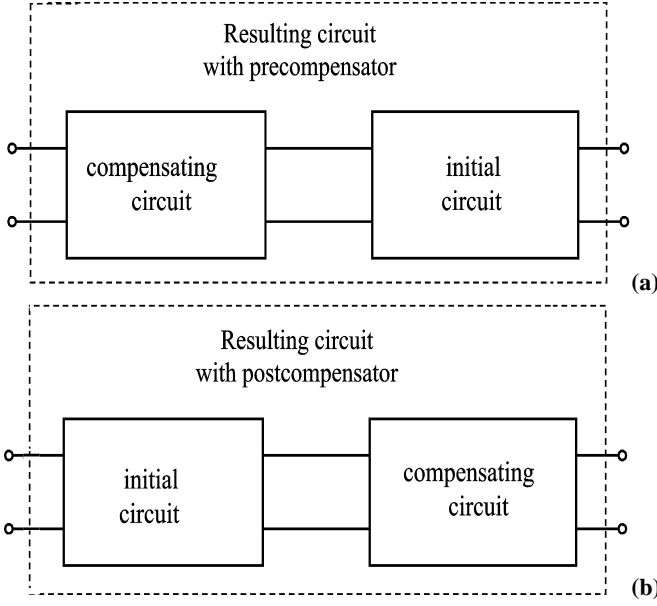
**Fig. 4.** Output voltage (a) and the amplitude spectrum (b) of the nonlinear circuit containing the LC-filter.

also necessary to take into account the significant sizes and weights of passive low-frequency filters.

From the above conclusions it follows that special protective means are required. In this paper we propose to use in particular synthesized compensators as input nonlinearities in electronic devices.

### 3 The compensation of nonlinearity

The synthesis of nonlinear compensators may be considered as the problem of the synthesis of operators connecting the sets of input and output signals of the considered devices. Such an approach assumes the following mathematical statement. Let  $X$  be the set of the circuit input signals  $x(t)$ ,  $Y$  be the corresponding set of the output signals  $y(t)$  of the initial



**Fig. 5.** Two cases of the connection of the compensator and the distorting circuit: with precompensator (a) and with postcompensator (b).

(distorting) circuit. Unique input-output relationship of the nonlinear circuit is described by the operator equation

$$\begin{aligned}
 y(t) &= V_1(p)x(t) + \sum_{k=2}^N V_k(p)x(t) = \\
 &= y_1(t) + \sum_{k=2}^N y_k(t), \quad \forall x(t) \in X, \forall y(t) \in Y, \quad (2)
 \end{aligned}$$

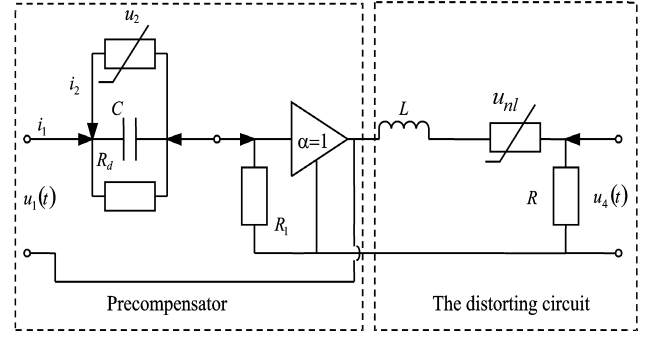
where  $V_1(p)$  is the linear operator of the initial circuit, and  $y_1(t)$  the corresponding output signal.  $V_k(p)$  is the uniform operator of power  $k$  causing the presence of the nonlinear component  $y_k(t)$  of power  $k$  in the output signal of the circuit.

The compensator synthesis problem lies in the construction of a nonlinear operator  $E(p)$  for the compensating circuit which acts in equation (2) in the following way (Schetzen, 1976):

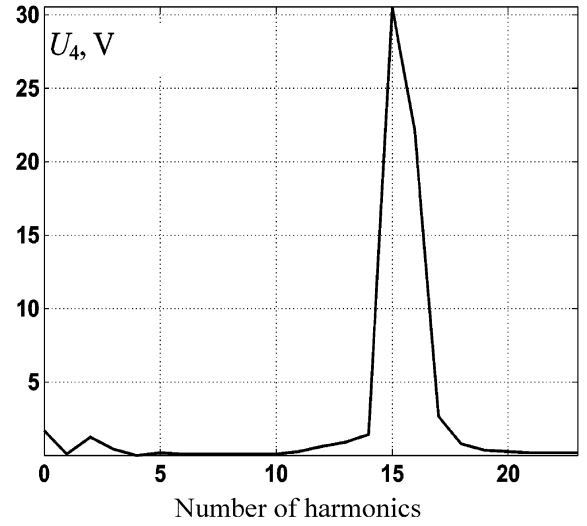
$$\begin{aligned}
 \tilde{x}(t) &= V_1(p)[E(p)x(t)] + \sum_{k=2}^N V_k(p)[E(p)x(t)] = \\
 &= Q(p)x(t), \quad (3)
 \end{aligned}$$

or

$$\begin{aligned}
 \tilde{x}(t) &= E(p)[V_1(p)x(t)] + E(p)\left[\sum_{k=2}^N V_k(p)x(t)\right] = \\
 &= Q(p)x(t). \quad (4)
 \end{aligned}$$



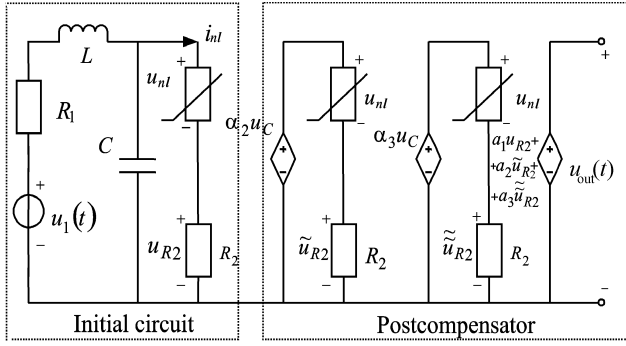
**Fig. 6.** The precompensator and the distorting circuit.



**Fig. 7.** The amplitude spectrum of the output voltage.

Here  $\tilde{x}(t)$  is the result of the compensation associated with the input signal of the initial circuit by a linear operator  $Q(p)$ . The series connection of the compensator is the usual way to connect it to an initial nonlinear circuit. Equation (3) is used to connect the compensator to the input of the initial circuit (Fig. 5). For the connection of the compensator to the output of the distorting circuit we use Eq. (4). In both cases we shall term the whole connection as the “resulting” one.

The aim of the compensation is to introduce the nonlinear operator  $E(p)$  so as to eliminate nonlinear components in the sums entering expressions (3), (4) and to create a resulting device with the linear operator  $Q(p)$ . Compensators are conveniently synthesized by using functional series and functional polynomials, since this mathematical apparatus allows one to form an analytical expression of the reaction of the resulting circuit and to select in it nonlinear components for the succeeding compensation. In this paper we used both approaches for the compensation. For the synthesis of the precompensator for the electrical circuit of Fig. 2 we used Picard’s series (Rugh, 1981), and the postcompensator was chosen for the electrical circuit with  $LC$ -filter based on



**Fig. 8.** The distorting circuit and postcompensator.

Volterra polynomials (Schetzen, 1976; Solovyeva, 1995).

Using Picard’s series for the construction of the pre-compensator for the nonlinear electric circuit presented in Fig. 2, we obtain the resulting connection represented in Fig. 6. The parameters of the linear part of the compensator are:  $C=10^{-5}$  nF,  $R_d=354.91$  kOhm,  $R_1=1000$  kOhm and those of the nonlinear element are:  $a_{22}=-0.493 \cdot 10^{-5}$ ,  $a_{32}=0.303 \cdot 10^{-8}$ . Accordingly the characteristic of the nonlinear element of the compensating circuit is  $i_2=4.93 \cdot 10^{-3} u_2^2 - 3.03 \cdot 10^{-6} u_2^3$  where  $i_2$  is measured in mA. The result of the compensation of the nonlinearity is shown in Fig. 7, which represents the amplitude spectrum of the output signal of the resulting circuit. Practically, the complete absence of lower harmonics in the amplitude spectrum of the output signal is obvious.

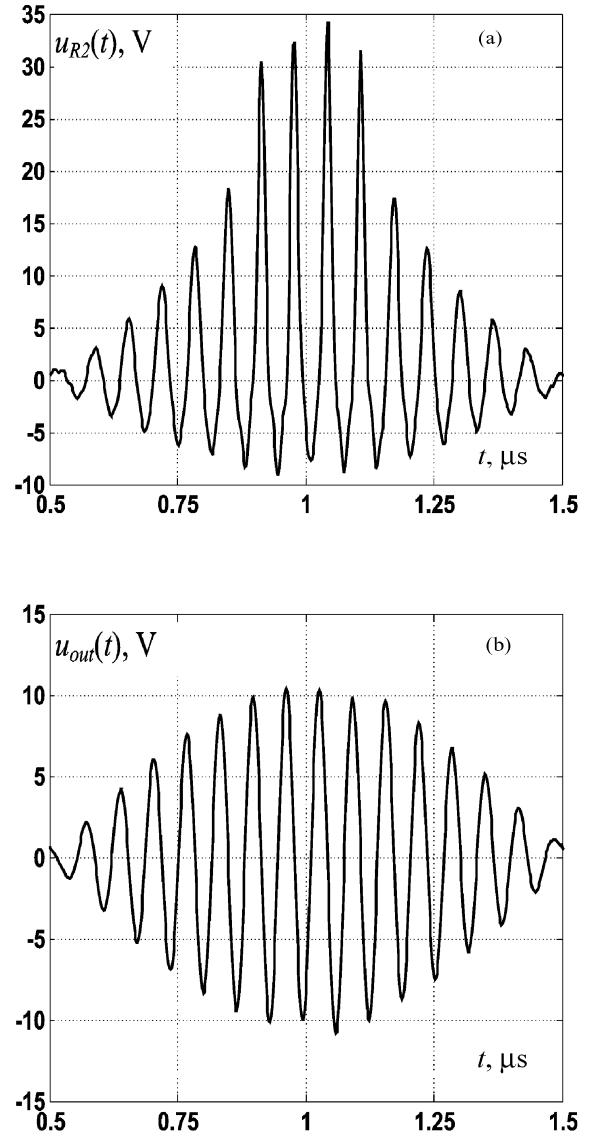
To construct the compensator for the circuit with the LC filter, we use the approach based on the Volterra series. The compensator was connected behind the initial nonlinear device. The resulting circuit is presented in Fig. 8, where  $\alpha_2=0.1$ ,  $\alpha_3=0.05$ ,  $a_1=0.0058$ ,  $a_2=-11.1111$ ,  $a_3=42.1053$ .

The results of compensation are shown in Figs. 9–10. The relation between the nonlinear element  $u_{nl}$  and  $i$  in the postcompensator is given by:  $u_{nl}=2718 \cdot i - 4932 \cdot i^2 + 3033 \cdot i^3$  where  $i$  is measured in mA,  $R_2=50$  kOhm. Figure 9 presents the output voltage  $u_{R2}(t)$  of the initial circuit, and the output voltage of the composed circuit. Figure 10 depicts the compensation error (the difference between the result of compensation and the reaction of the linear RLC-circuit) and the amplitude spectrum of the output signal of the composed circuit.

The analysis of Figs. 9–10 shows, that the synthesized compensator provides a high quality of noise suppression.

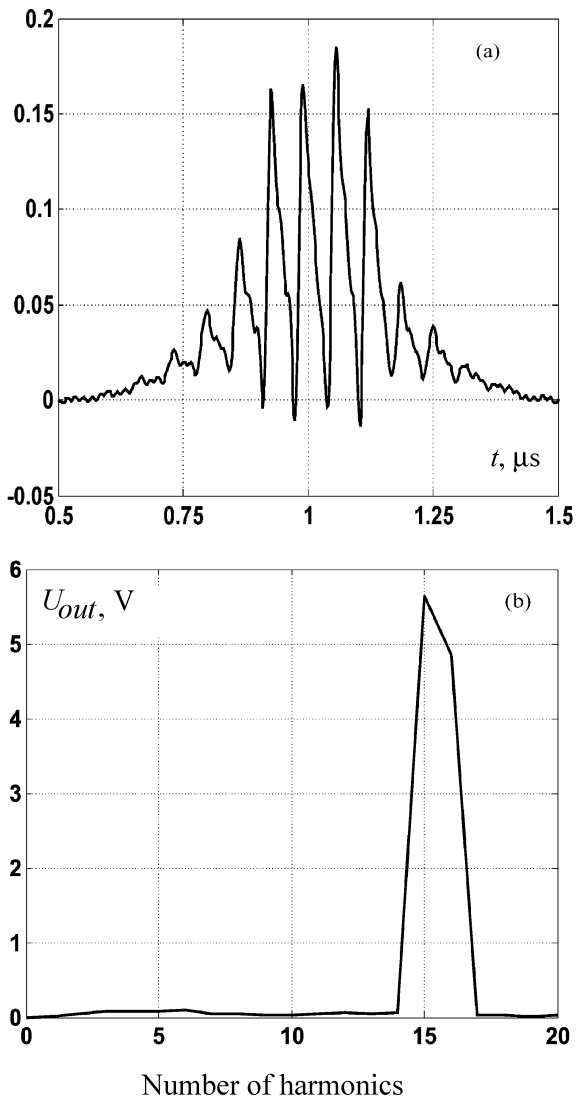
#### 4 Conclusion

In this paper we have shown that a high frequency excitation of a system can be converted to a low frequency one, due to the interior features of the system itself. Nonlinearities are necessary for this high frequency- to low frequency- energy conversion. Beats or amplitude modulated signals with



**Fig. 9.** The voltage of the output of the initial circuit (a), and the voltage of the output signal of the circuit with postcompensator (b).

high frequency carriers are demodulated to those signals which contain remarkable low-frequency amplitudes in their spectra. These low-frequency signals are unwanted and may cause malfunctions of the considered electronic system since operational frequencies of the system lie in this frequency band. Therefore, protection measures against low-frequency excitations are required. We describe in the previous sections that this is not an easy undertaking and has to be adapted to the individual system. A pre- or postcompensator appears to be a reasonable protection unit. This is an additional nonlinear circuit which is brought into the system to compensate the unwanted low-frequency noise. In order to estimate this protection circuit advanced mathematical procedures (Picard’s and Volterra series expansions) have to be applied. The action of this protection mean is convincing and should be considered in the context of EMC measures.



**Fig. 10.** The error of the compensation (a), and the amplitude spectrum of the output voltage (b).

*Acknowledgements.* This work was supported by the state Saxony-Anhalt under the project number FKZ:3330A/0021T.

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