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Computation of the frequency response of a nonlinearly loaded antenna within a cavity

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Abstract. We analyze a nonlinearly loaded dipole antenna which is located within a rectangular cavity and excited by an electromagnetic signal. The signal is composed from two different frequencies. In order to calculate the spectrum of the resulting electromagnetic field within the resonator we transform the antenna problem into a network problem. This requires to precisely determine the antenna impedance within the cavity. The resulting nonlinear equivalent network is solved by means of the harmonic balance technique. As a result the occurrence of low intermodulation frequencies within the spectrum is verified.

1 Introduction

In the framework of EMC analysis of complex systems the electromagnetic coupling path from EMI-source to EMI-victim usually is divided into an exterior and an interior problem (Lee, 1995; Tesche et al., 1997). The interior problem involves the electromagnetic coupling to an EMI-victim within a resonator. Its analytic description mathematically is rather complicated, in particular if electromagnetic coupling to nonlinear elements is involved. From an EMC engineering point of view it is important to determine the resulting frequency spectrum within the resonator. From this it is possible to estimate potential upset or damage due to the excitation of low-frequency cavity resonances that might occur after demodulation of an incoming electromagnetic signal at a nonlinear element.

Nonlinearly loaded antennas in free space already have been investigated in considerable detail (Sarkar and Weiner, 1976; Liu and Tesche, 1976; Kanda, 1980). A working approach is to convert the electromagnetic field problem to a corresponding nonlinear network problem and to solve the latter one by the harmonic balance technique (Huang and Chu, 1993; Lee, 2000). The analysis of the nonlinear coupling problem within a cavity follows, in principle, the same pattern as the analysis in free space. To properly describe the electromagnetic coupling the Green's function of the cavity, which usually is given in terms of a series, has to be used in place of the Green's function of free space. Then it is most important to use a representation of the cavity's Green's function which is quickly convergent in order to ensure numerical efficiency and accuracy during its evaluation. This must be guaranteed for evaluation both close to the source region and close to a resonance. In the present paper we will use a specific representation which is based on the Ewald sum technique (Gronwald, 2003).

The paper is organized as follows: In Sect. 2 we establish the formulation of the problem and collect some equations. Sect. 3 focuses on the determination of the equivalent network problem and reviews as solution procedure the reflection algorithm which is a special case of the harmonic balance technique (Maas 1988). Results for a specific example are provided by Sect. 4 and followed by a conclusion.

2 Setup and network formulation of the problem

As a model of a nonlinear electric or electronic component which is susceptible to an exciting electromagnetic field we take a nonlinearly loaded dipole antenna which is placed within a rectangular cavity, compare Fig. 1. We excite the antenna by an electromagnetic source field and want to calculate the frequency response, i.e., the spectrum of the resulting electromagnetic field that, in turn, is determined by the spectrum of the resulting antenna current.

To solve this field theoretical problem for the unknown antenna current we transform it into a network problem. The Thévenin and Norton equivalent of the dipole antenna are shown in Fig. 2. They contain the equivalent current source $I_{\text{eq}}(\omega)$ and the equivalent voltage source $V_{\text{eq}}(\omega)$, respectively, that are present at the antenna input terminal and due to the electromagnetic source field. They also involve the antenna input admittance $Y(\omega)$ and input impedance $Z(\omega)$,

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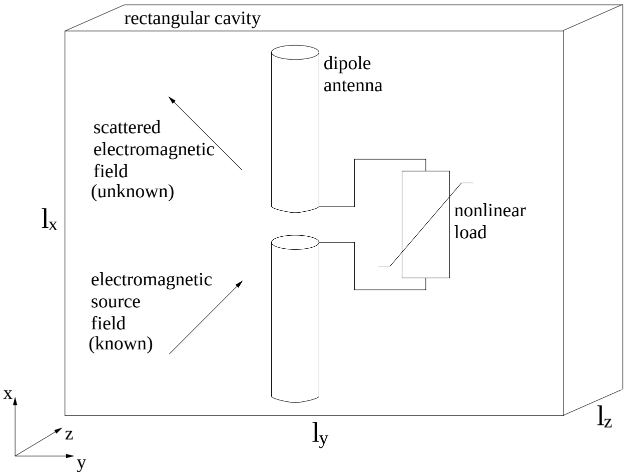

Fig. 1. Setup of the problem: A nonlinearly loaded antenna is placed within a rectangular cavity of dimensions l_x, l_y, l_z, and excited by a known electromagnetic field. The frequency spectrum of the scattered electromagnetic field needs to be determined.

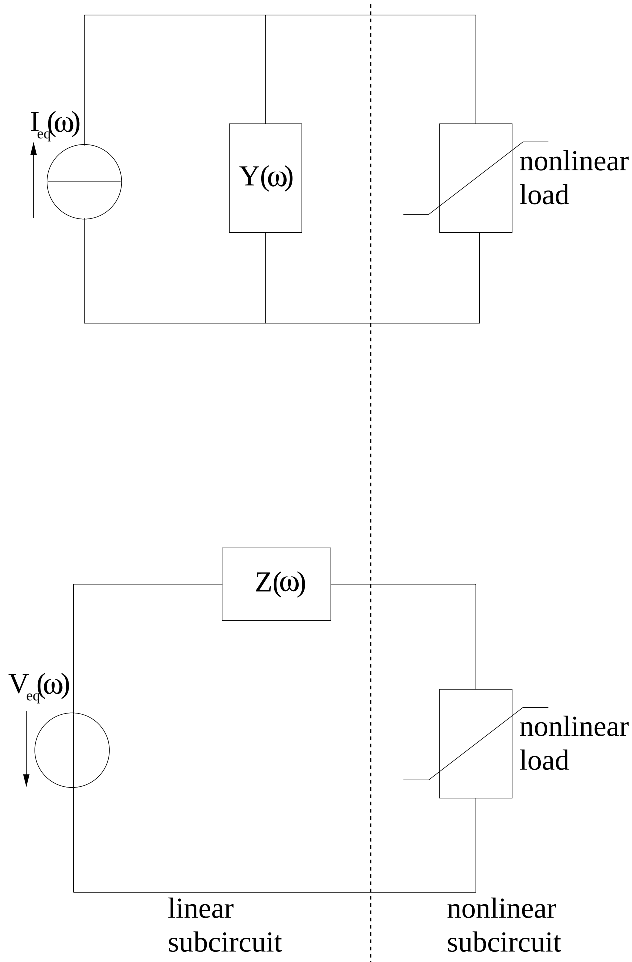

Fig. 2. Equivalent network formulation of the problem in Fig. 1.

respectively. We note that $Y(\omega)$ and $Z(\omega)$ are characterized not only by the antenna geometry but also by the dimensions of the rectangular cavity and by the antenna position and orientation within the cavity. To precisely determine the parameters $I_{\rm eq}(\omega)$, $Y(\omega)$, or likewise $V_{\rm eq}(\omega)$, $Z(\omega)$, is a demanding field theoretical task. We will use, as an approximate solution scheme, the method of moments and take advantage of the cavity's Green's function. Once the equivalent parameters are known we can use standard methods to solve for the antenna current. The details for the two step procedure

1. determination of the equivalent network parameters by the method of moments and

2. solving the network problem for the antenna current by the reflection algorithm

will be given in the following section.

3 Solution procedure

3.1 Method of moments within a rectangular cavity

There are two standard integral equations which can be used to determine the current on a linear wire antenna. These are Pocklington's equation and Hallén's equation (Nakano, 1996). For the following we choose Hallén's equation in frequency domain with time dependency $\exp(j\omega t)$.

We consider a straight dipole antenna which is aligned with the x-axis and position its center at $x{=}l_x/2$. To explicitly write down Hallén's equation requires to explicitly specify the electromagnetic source field. For the method of moments procedure this specific choice is not really important since it merely determines the inhomogeneous part of the corresponding algebraic system of equations which normally poses no computational difficulties. The important and demanding element of the calculation is the computation of the homogeneous part, i.e., the matrix elements of the algebraic system, and this does not involve the electromagnetic source.

We choose as excitation a slice generator that is characterized by $E_x^q(x){=}V_0\delta(x{-}l_x/2)$. If furthermore a thin-wire approximation is employed it is found that Hallén's equation acquires the form

$$\int_{-L/2}^{L/2} G^A_{\mathrm{cav}xx}(x,x')I(x')dx' = -\frac{j}{\eta}\Big(A\cos(k(x-\frac{l_x}{2}))+\frac{V_0}{2}\sin(k|x-\frac{l_x}{2}|)\Big)\,, \qquad (1)$$

with $G^A_{\mathrm{cav}xx}$ the xx-component of the vector potential's dyadic Green's function of the cavity. The symbol η denotes the intrinsic impedance of the surrounding medium, $\eta{=}\sqrt{\mu/\varepsilon}$, and the wavenumber is given by $k{=}\omega/c$. Since we have in mind to solve Hallén's equation within a lossy cavity we will pass to a complex wavenumber via $k{\to}k(1{-}j/(2Q))$, where Q denotes the quality factor of the cavity. The integral extends over the length of the antenna which is denoted by L and the antenna radius is denoted by

ρ. In Eq. (1) the unknowns are the current distribution $I(x)$ and the integration constant A. These unknowns are to be determined by the method of moments.

It was shown (Gronwald, 2003) that $G^A_{\mathrm{cav}xx}$ can be written in the form of a specific ray-mode representation,

$$G^A_{\mathrm{cav}xx} = \underbrace{G^A_{\mathrm{cav}xx1}}_{\text{mode part}} + \underbrace{G^A_{\mathrm{cav}xx2}}_{\text{ray part}} \tag{2}$$

where

$$G^A_{\mathrm{cav}xx1} = \frac{\mu}{8l_xl_yl_z}\sum_{m,n,p=-\infty}^{\infty}\sum_{i=0}^{7} A_i^{xx}\times \frac{\exp\left(-\frac{k_0^2-k^2}{4E^2}\right)}{k_0^2-k^2}\exp\big(j(k_{0x}X_i+k_{0y}Y_i+k_{0z}Z_i)\big)\,, \tag{3}$$

$$G^A_{\mathrm{cav}xx2} = \frac{\mu}{8\pi}\sum_{m,n,p=-\infty}^{\infty}\sum_{i=0}^{7} A_i^{xx}\times \left[\frac{\exp(jkR_{i,mnp})\mathrm{erfc}(R_{i,mnp}E+jk/2E)}{R_{i,mnp}}+ \frac{\exp(-jkR_{i,mnp})\mathrm{erfc}(R_{i,mnp}E-jk/2E)}{R_{i,mnp}}\right]. \tag{4}$$

Here the coefficient A_i^{xx} is defined by

$$A_i^{xx} = \begin{cases} +1\,, & i=0,3,4,7 \\ -1\,, & i=1,2,5,6 \end{cases}. \tag{5}$$

The length $R_{i,mnp}$ represents the distance between a source at $\boldsymbol{r}'=(x',y',z')$ and its mirror sources to an observation point at $\boldsymbol{r}=(x,y,z)$. It is given by

$$R_{i,mnp} = \sqrt{\rho^2+(X_i+2ml_x)^2+(Y_i+2nl_y)^2+(Z_i+2pl_z)^2} \tag{6}$$

with

$$X_i = \begin{cases} x-x'\,, & i=0,1,2,3 \\ x+x'\,, & i=4,5,6,7 \end{cases}, \tag{7}$$

$$Y_i = \begin{cases} y-y'\,, & i=0,1,4,5 \\ y+y'\,, & i=2,3,6,7 \end{cases}, \tag{8}$$

$$Z_i = \begin{cases} z-z'\,, & i=0,2,4,6 \\ z+z'\,, & i=1,3,5,7 \end{cases}. \tag{9}$$

The components k_{0x}, k_{0y}, and k_{0z} are defined by the vector $\boldsymbol{k}_0$ according to

$$\boldsymbol{k}_o := (k_{0x},k_{0y},k_{0z}) := (m\pi/l_x, n\pi/l_y, p\pi/l_z)\,, \tag{10}$$

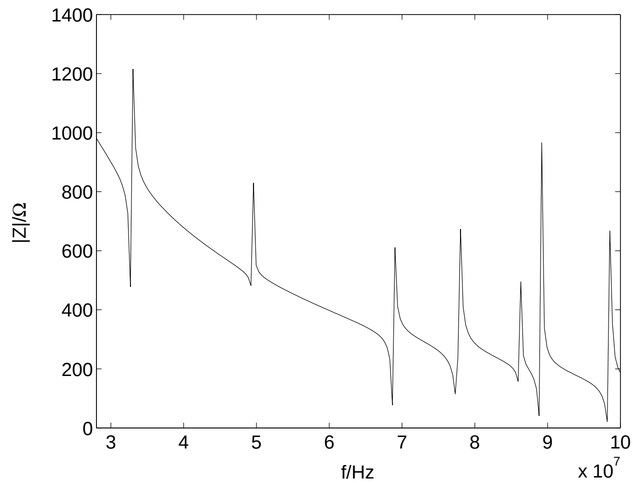

Fig. 3. Absolute value of the input impedance of a dipole antenna within a rectangular cavity. It was obtained by the method of moments as described in the text. The peaks correspond to resonances of the cavity.

and E denotes an adjustable parameter. Finally, we introduced in (4) the complementary error function erfc(z).

The benefit of the ray-mode representation (3), (4) is its quick convergence in both the source region $\boldsymbol{r}\to\boldsymbol{r}'$ and close to resonance. This feature is typical for this kind of representation (Felsen, 1984). The number of terms required for convergence can be of orders of magnitude smaller as compared to a standard mode or ray representation (Gronwald et al., 2002).

The solution of Hallén's equation (1) yields the current distribution on the dipole antenna within the cavity (Gronwald, 2003). From this the input impedance $Z(\omega)$ or input admittance $Y(\omega)$ can directly be obtained since $V_{\mathrm{eq}}(\omega)$ already is given by the exciting slice generator. In Figure 3 we show as an illustration the input impedance of a 1m dipole antenna which is contained within a cavity of dimensions 7m$\times$6m$\times$3m and constant quality factor Q=1000. The dimensions correspond to a real life mode-stirred chamber, as it is installed at the University of Magdeburg, for example.

3.2 Reflection algorithm

The nonlinear network of Fig. 2 can mathematically be described within the framework of the harmonic balance technique. For a solution of the corresponding harmonic balance equations the reflection algorithm can be used (Kerr, 1975; Maas, 1988). The reflection algorithm is similar to the Bergeron method (Magnusson et al., 2001) and imitates a turn-on process where the source is switched on at a specific moment. In order to have a dynamical process that, hopefully, will settle after some time into a stable state of the network, one introduces a fictitious, ideal transmission line between the linear and the nonlinear subcircuit of Fig. 2, compare Fig. 4. The transmission line is supposed to not alter the steady state properties of the complete nonlinear circuit. This

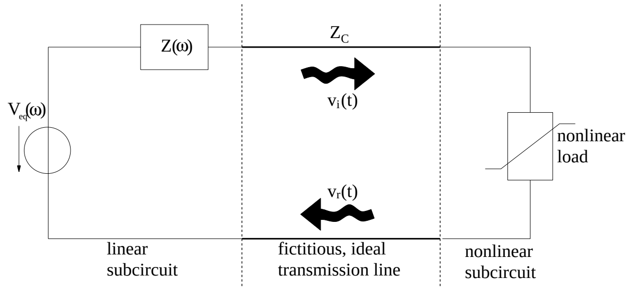

Fig. 4. The reflection algorithm conceptually requires to insert an ideal transmission line between the linear and nonlinear subcircuit. After the turning on of the source $V_{\mathrm{eq}}(\omega)$ there will be an incident wave $v_i(t)$ which propagates along the transmission line towards the nonlinear load. The wave will interact with the nonlinear load and, subsequently, form a reflected wave $v_r(t)$ that travels back towards the linear subcircuit. There it will be reflected again to form a new incident wave that propagates towards the nonlinear load. The process continues until a steady state is reached.

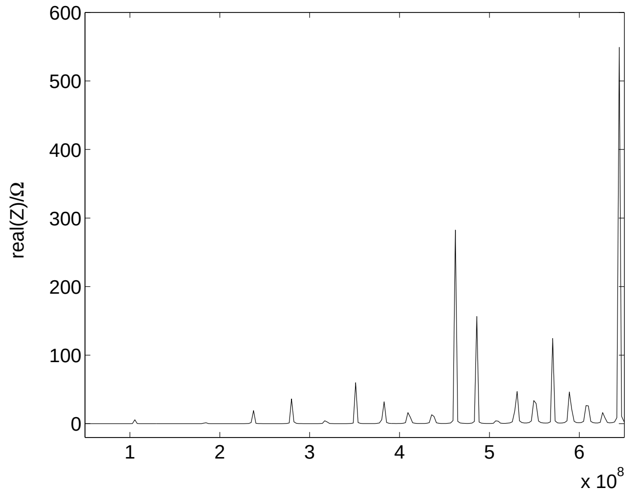

Fig. 5. Real part of the input impedance $Z(\omega)$. It characterizes the antenna-cavity configuration of the specific example described in the text.

implies that its length must be an integer number of wavelengths of the exciting frequencies. In the case of two exciting frequencies f_1 and f_2 with corresponding wavelengths λ_1 and λ_2, respectively, it follows that the wavelengths must fulfill $\lambda_1/\lambda_2=n_2/n_1$ for some integers n_1 and n_2.

The reflection algorithm is established by the following steps:

1. We first calculate the initial incident wave $v_i^0(t)$. If the impedance of the ideal transmission line, which, incidentally, is real, is denoted by Z_C the initial wave will be of the form

$$v_i^0(t) = \frac{|V_{eq}|Z_C\cos(\omega t+\theta)}{\sqrt{|Z|^2+Z_C^2}}\,, \tag{11}$$

$$\theta := \arctan\Big(\frac{\mathrm{Im}(Z)}{\mathrm{Re(Z)}+Z_C}\Big)\,. \tag{12}$$

In case of two exciting frequencies the incident wave is a superposition of two such contributions.

2. The propagation along the transmission line is ideal and does not change the waveform. At the nonlinear load we need to solve a nonlinear equation for the current

$$i^0(t) = f(v_i^0(t), i^0(t))\,. \tag{13}$$

The explicit nonlinear equation is determined by the nonlinear load, of course.

3. Now the reflected wave is constructed in accordance to usual transmission line theory. This yields

$$v_r^0(t) = v_i^0(t) - Z_C i^0(t)\,. \tag{14}$$

4. To describe the subsequent reflection at the linear subcircuit we need to Fourier transform $v_r^0(t)$ to find its sampled frequency components $\tilde{V}_r^0(\omega_k)$. Then the new incident wave is given by,

$$v_i^1(t) = v_i^0(t) + \frac{1}{2}\sum_{k=-K}^{K}\Gamma_{\omega_k}\tilde{V}_r^0(\omega_k)\exp(j\omega_k t)\,, \tag{15}$$

where the reflection coefficient Γ_{ω_k} follows from

$$\Gamma_{\omega_k} = \frac{Z(\omega_k)-Z_C}{Z(\omega_k)+Z_C}\,. \tag{16}$$

One should note that

$$\Big(\Gamma_{\omega_k}\tilde{V}_r^0(\omega_k)\Big)^* = \Gamma_{\omega_{-k}}\tilde{V}_r^0(\omega_{-k}) \tag{17}$$

since $Z(\omega_{-k})=Z^*(\omega_k)$ and thus $\Gamma_{\omega_{-k}}=\Gamma^*_{\omega_k}$. Also we have $\tilde{V}_r^0(\omega_{-k})=(\tilde{V}_r^0(\omega_k))^*$ since these Fourier coefficients have been obtained from a real time signal. Therefore, the updated time signal in Eq. (15) is a real function, as well.

5. Now we return to step 2, with $v_i^0(t)$ replaced by $v_i^1(t)$, until the process has converged. After convergence we read off the spectrum of the current $i(t)$ through the nonlinear load which equals the antenna current at the input terminals.

4 Example

We now consider, as an example, a specific configuration: We choose a rectangular cavity of dimensions $l_x=l_y=l_z=2$m with its center at position (1, 1, 1)m. As quality factor we take the fixed value $Q=1000$. We further assume a dipole

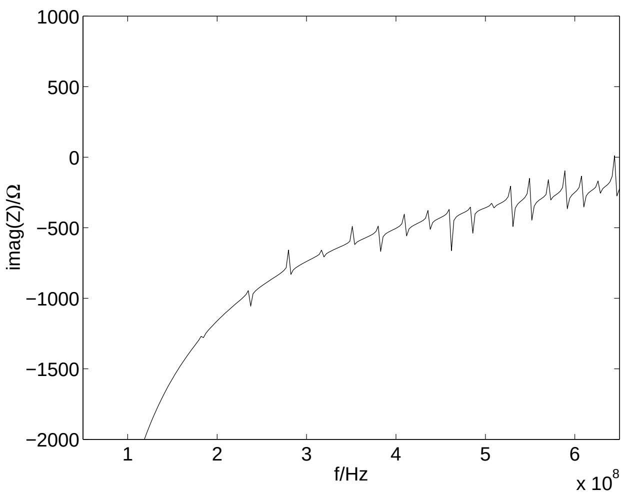

Fig. 6. Imaginary part of the input impedance $Z(\omega)$ of the antenna-cavity configuration.

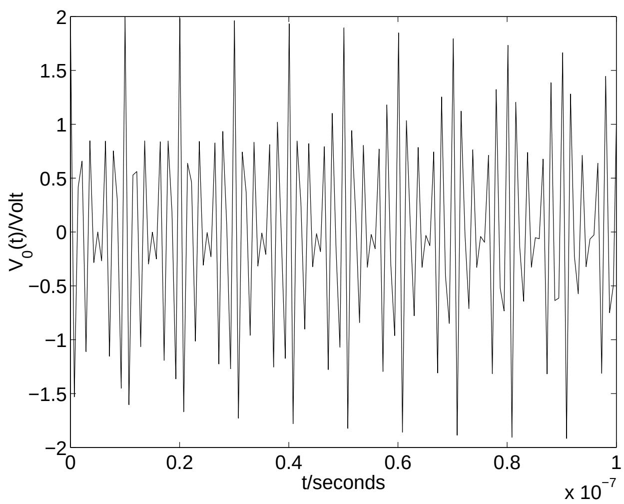

Fig. 7. The exciting signal $v_0(t)=1\mathrm{V}(\sin(2\pi f_1 t)+\sin(2\pi f_2 t))$ which is applied to the input terminals of the nonlinearly loaded antenna.

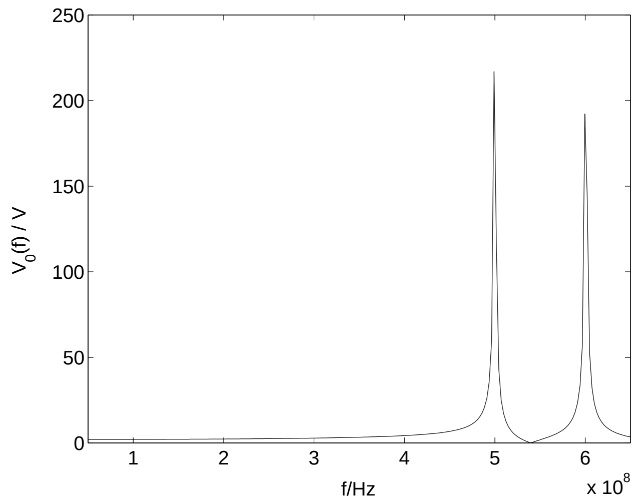

Fig. 8. Spectrum $\tilde{V}_0(f)$ of the signal $v_0(t)$ in Fig. 7, as obtained by a Fast Fourier Transform with 512 sample points. Displayed are the sampled values which, incidentally, have the same physical dimension as the original signal.

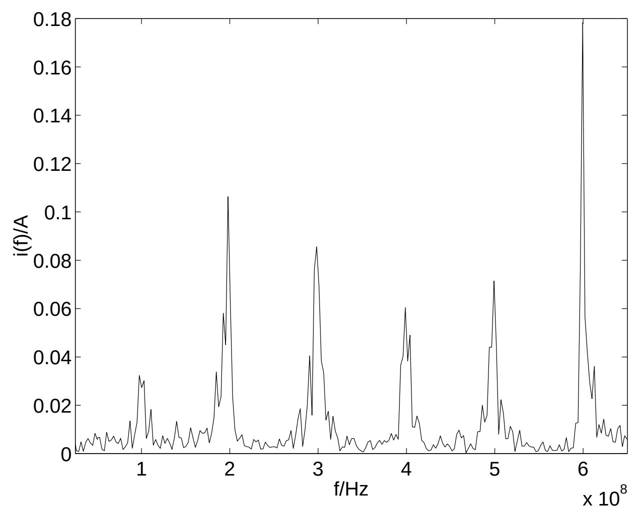

Fig. 9. Resulting spectrum of the antenna current after 15 iterations. Further iterations within the reflection algorithm only lead to minimal changes.

antenna of length L=0.15 m and radius ρ=10^{-3} m which is aligned with the x-axis and has its center at x=1 m. The y- and z-coordinates of the antenna are set to 1 m. For this antenna the real and imaginary part of the input impedance, as calculated by the method of moments, is displayed in Fig. 5 and Fig. 6, respectively.
As nonlinear load we choose a diode with v−i-characteristic

$$i(t) = i_{\mathrm{sat}}\big(\exp(v(t)/v_c) - 1\big)\,, \tag{18}$$

where i_{sat}=10 nA and v_c=25 mV. The antenna is excited by a slice generator with frequencies f_1=500 MHz and f_2=600 MHz of amplitude V_0=$|V_{\mathrm{eq}}|$=1 V. This signal is shown in time and frequency domain in Fig. 7 and Fig. 8, respectively.

With these specifications we can enter the reflection algorithm and calculate the initial incident wave $v_i^0(t)$ according to Eqs. (11), (12), where we have chosen Z_C=50 Ω. We note that according to transmission line theory the voltage at the nonlinear load, i.e., the voltage at the end of the transmission line, is given by $2v_i^0(t) - Z_C i^0(t)$. Therefore, the nonlinear equation to be solved in the second step, compare (13), explicitly reads

$$i^0(t) = i_{\mathrm{sat}} \exp\big((2v_i^0(t) - Z_C i^0(t))/v_c - 1\big)\,. \tag{19}$$

After having solved Eq. (19) for the current $i^0(t)$ it is trivial to calculate the reflected wave $v_r^0(t)$. To construct the new incident wave according to Eq. (15) we use a Fast Fourier Transformation of $v_r^0(t)$ with 512 sample points.

As can be expected, already $v_r^0(t)$ contains intermodulation frequencies. The subsequent reflections exhibit satisfactory convergence towards a specific spectrum of the antenna current. In Fig. 9 the spectrum obtained after 15 iterations is shown. Apart from numerical noise the intermodulation frequencies at 100 MHz, 200 MHz, 300 MHz, and 400 MHz clearly appear.

5 Conclusions

We have demonstrated how to calculate the frequency spectrum of a nonlinearly loaded thin-wire antenna if excited within a rectangular cavity. In view of EMC analysis the results imply the following: In general, high frequency signals can couple through small apertures into resonating environments that act like cavities. In the presence of nonlinear elements these signals can generate, in turn, low intermodulation frequencies that might coincide with a cavity's resonance frequency. The type of intermodulation frequencies is determined, in the first place, by the type of the exciting signals and the type of nonlinearity. Our simulations have indicated that, apart from a minor shift due to the damping within the resonator, properties of the cavity are not reflected in the position of the spectral components. However, they are reflected in their amplitudes.

It is important to note that the resulting spectrum of Fig. 9 relates to the antenna current. From this current the corresponding electromagnetic field can be calculated after weighting with the cavity's Green's function and integration along the antenna. It is at this point where the cavity's resonances can lead to a major amplification of the electromagnetic coupling since, at resonance and depending on the quality factor of the cavity, the cavity's Green's function will become dominant if compared to the situation of free space.

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