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Review of singular potential integrals for method of moments solutions of surface integral equations

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Abstract. Accurate evaluation of singular potential integrals is essential for successful method of moments (MoM) solutions of surface integral equations. In mixed potential formulations for metallic and dielectric scatterers, kernels with $1/R$ and $\nabla 1/R$ singularities must be considered. Several techniques for the treatment of these singularities will be reviewed. The most common approach solves the MoM source integrals analytically for specific observation points, thus regularizing the integral. However, in the case of $\nabla 1/R$ a logarithmic singularity remains for which numerical evaluation of the testing integral is still difficult. A recently by Ylä-Oijala and Taskinen proposed remedy to this issue is discussed and evaluated within a hybrid finite element – boundary integral technique. Convergence results for the MoM coupling integrals are presented where also higher-order singularity extraction is considered.

1 Introduction

Exact solutions of electromagnetic antenna and scattering problems often rely on integral equations being solved by the method of moments (MoM) with Galerkin's method, using triangular Rao-Wilton-Glisson (RWG) vector basis functions (Rao et al., 1982). In mixed potential formulations for metallic and dielectric scatterers, the kernels of surface integrals include the Green's function of free space, as well as the gradient of Green's function. Thus, singularities of order $1/R$ and $\nabla 1/R$ must be considered, where $R=|\mathbf{r}-\mathbf{r}'|$ is the distance between observation and source points. Because of these terms, the surface integrals become singular if a testing point $\mathbf{r}$ is near the source element. In order to calculate the singular surface integrals, special methods must be used, because numerical integration routines lead to inaccurate solutions. There are many methods that can be used to evaluate singular potential integrals, such as the Duffy's transformation (Duffy, 1982; Chew et al., 2001) and the singularity extraction method (Wilton et al., 1984; Graglia, 1993; Eibert and Hansen, 1995).

In Duffy's transformation, the source triangle is divided into three subtriangles, with common vertex at the singular point. Then, the integrals over each subtriangle are transformed into integration over a square. This procedure cancels the singularity. Duffy's transformation has three main disadvantages. First, it is accurate only for sufficiently regular triangles. Second, new integration points on the source integral have to be generated for each integration point on the testing integral. This increases the computation time. Third, singularities of order $\nabla 1/R$ cannot be easily evaluated, because Duffy's transformation is derived for functions having a point singularity of order $1/R$.

In the singularity extraction method, surface integrals are regularized by extracting the singular term from the Green's function. The inner source integral of the extracted term is calculated analytically in primed coordinates for specific observation points. The remaining function is regular and the outer testing integral can be calculated numerically in unprimed coordinates. However, after the extraction of the singular term, the remaining function is not necessarily continuously differentiable, which means that a straightforward application of a numerical integration routine may lead to an inaccurate solution. Further, difficulties in integration may occur in the case of $\nabla 1/R$, when the source and test triangles have common points and are not in the same plane. In particular, after extracting the singularity and calculating the inner source integral analytically, a logarithmic singularity remains on the outer testing integral. Therefore, if higher accuracy is required, the testing integral cannot be calculated by a standard numerical calculation technique.

Ylä-Oijala and Taskinen recently proposed a remedy to this issue (Ylä-Oijala and Taskinen, 2003). The surface integrals are additionally regularized, by extracting more terms from the Green's function and its gradient. The additional extracted terms are integrated analytically over the source triangle in primed coordinates. The remaining function is at least

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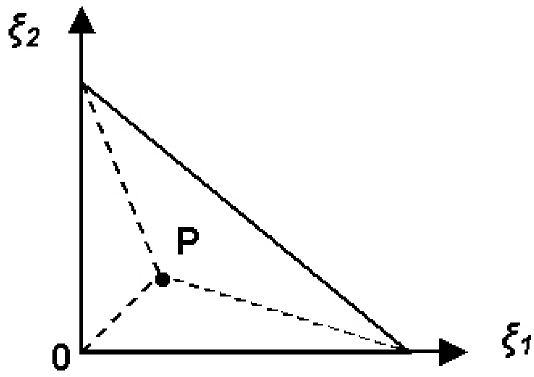

Fig. 1. Division into subtriangles.

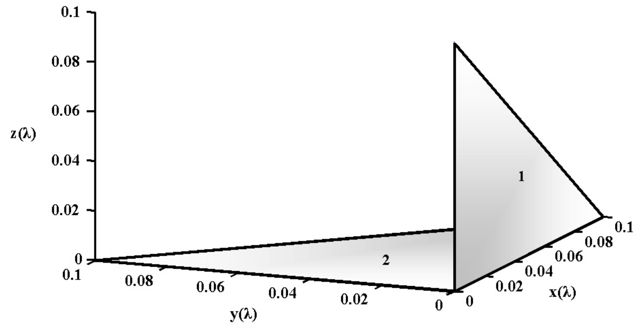

Fig. 2. Triangles for numerical examples.

once continuously differentiable and is integrated over the testing triangle numerically in unprimed coordinates. Further, in evaluating the integrals including the gradient of the Green's function, the source integral of the surface component of the singular term $\nabla 1/R$ is transformed into a line integral and the order of integration is being changed. Now, the inner testing integral is evaluated analytically in unprimed coordinates and the outer source line integral is evaluated by a standard numerical calculation technique. By these modifications logarithmic singularity can be avoided. All singularities are being extracted and calculated in closed form and numerical integration is applied only for regular and continuously differentiable functions. In this work, the above method has been evaluated within a hybrid Finite Element – Boundary Integral (FEBI) technique, using Combined Field Integral Equation (CFIE). Convergence results of the MoM coupling integrals are presented for perpendicular source and testing triangles with common edge. Using this new method, convergence of singular integrals is achieved with significantly less integration points and, because of this, accuracy, robustness and computation time of the FEBI technique is improved.

2 CFIE formulation

Consider the problem of electromagnetic scattering or radiation including dielectric and arbitrarily shaped three-dimensional bodies. According to Huygen's principle, the electric and magnetic field can be expressed as a function of the equivalent current densities on the surface S of the scatterers and the Green's function of the electromagnetic problem. Then, applying the boundary condition of the surface current densities for observation points on the surface S, integral equations can be derived with unknown quantities the tangential components of the electric and magnetic field (or equivalently the densities of the Huygen's surface currents).

For the equivalent magnetic current density $\mathbf{M}{=}\mathbf{E}{\times}\mathbf{n}$ on S, the Electric Field Integral Equation (EFIE) in mixed potential formulation is derived:

$$\begin{aligned}\frac{1}{2}(\mathbf{n}(\mathbf{r}))\times\mathbf{M}_S(\mathbf{r}) = & -j\frac{\omega\mu}{4\pi}\int\!\!\int_{S'}\mathbf{J}_S(\mathbf{r}')G(\mathbf{r},\mathbf{r}')dS' \\ & -j\frac{1}{4\pi\omega\varepsilon}\nabla\int\!\!\int_{S'}\nabla'_S\cdot\mathbf{J}_S(\mathbf{r}')G(\mathbf{r},\mathbf{r}')dS' \\ & -\frac{1}{4\pi}\int\!\!\int_{S'}\nabla G(\mathbf{r},\mathbf{r}')\times\mathbf{M}_S(\mathbf{r}')dS'+\mathbf{E}^{\mathrm{inc}}(\mathbf{r}),\end{aligned} \tag{1}$$

with $\mathbf{r}\in S'$. Likewise, for the equivalent electric current density $\mathbf{J}{=}\mathbf{n}{\times}\mathbf{H}$ on S the Magnetic Field Integral Equation (MFIE) in mixed potential formulation is derived:

$$\begin{aligned}\frac{1}{2}\mathbf{J}_S(\mathbf{r}') = & -j\frac{\omega\mu}{4\pi}\mathbf{n}(\mathbf{r})\times\int\!\!\int_{S'}\mathbf{M}_S(\mathbf{r}')G(\mathbf{r},\mathbf{r}')dS' \\ & -j\frac{1}{4\pi\omega\mu}\mathbf{n}(\mathbf{r})\times\nabla\int\!\!\int_{S'}\nabla'_S\cdot\mathbf{M}_S(\mathbf{r}')G(\mathbf{r},\mathbf{r}')dS' \\ & -\frac{1}{4\pi}\mathbf{n}(\mathbf{r})\times\int\!\!\int_{S'}\nabla G(\mathbf{r},\mathbf{r}')\times\mathbf{J}_S(\mathbf{r}')dS'+\mathbf{n}(\mathbf{r})\times\mathbf{H}^{\mathrm{inc}}(\mathbf{r}),\end{aligned} \tag{2}$$

with $\mathbf{r}\in S'$. $\mathbf{E}^{\mathrm{inc}}$, $\mathbf{H}^{\mathrm{inc}}$ is the incident field, $G(\mathbf{r},\mathbf{r}') = e^{-jkR}/R$ is the Green's function of free space outside the scatterer, $\mathbf{n(r)}$ is the normal unit vector of surface S and $\nabla'_S\cdot$ denotes the surface divergence with respect to the prime coordinates.

The CFIE is the following linear combination of the EFIE and the MFIE:

$$Z_0(1-\alpha)\,\mathrm{MFIE}+\alpha\mathrm{EFIE}=0, \tag{3}$$

where α is the combination parameter, with $0\leq\alpha\leq 1$, and Z_0 is the characteristic impedance of free space. Using CFIE, the internal resonance problem, which produces incorrect components in the field solution, can be avoided.

The integral equations are solved by the method of moments. Within the hybrid FEBI technique, the volume of the scatterer is modelled with tetrahedrons, which leads to a surface model with triangular elements. The surface currents are described by RWG functions (Rao et al., 1982) and the

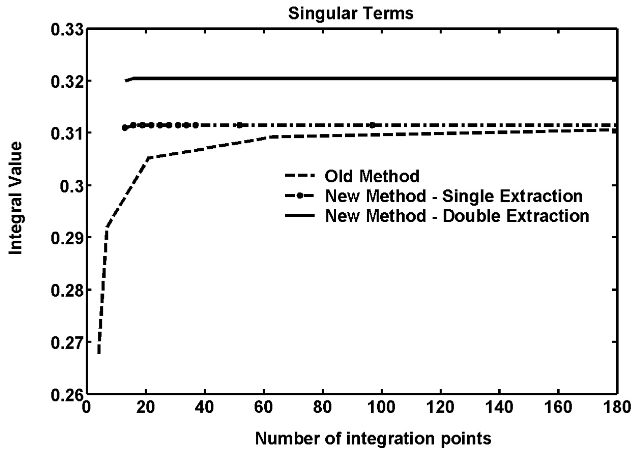

Fig. 3. Convergence of singular terms of EFIE coupling integrals with singularity $\nabla 1/R$ using the new method with single and double singularity extraction, compared with the corresponding values using the old method.

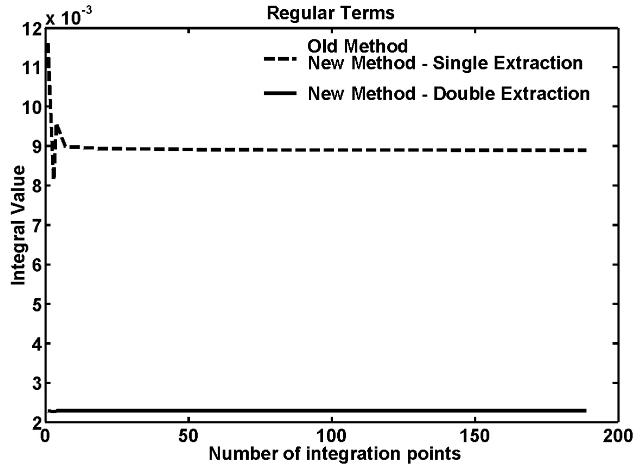

Fig. 4. Convergence of regular terms of EFIE coupling integrals with singularity $\nabla 1/R$ using the new method for single and double extraction, compared with the corresponding values using the old method.

Galerkin's method is used for the testing procedure. This means that the unknown surface currents have the form

$$\mathbf{J}_S=\sum_{n=1}^{N} i_n \boldsymbol{\beta}_n,\ \ \mathbf{M}_S=-\sum_{n=1}^{N} u_n \boldsymbol{\beta}_n. \tag{4}$$

The testing procedure is applied with RWG functions, which means that $\boldsymbol{\beta}_m=\boldsymbol{\beta}_n$. After applying this solution to the integral equation, various surface integrals must be solved. In the next section these integrals are presented and several techniques for the treatment of these integrals in singular cases are reviewed.

3 Singular integral treatments

An application of the MoM with the Galerkin's method in order to solve the CFIE using RWG basis functions requires calculation of the following integrals for the EFIE part:

$$I_1^{\mathrm{EFIE}}=\int_S\int \boldsymbol{\beta}_m(\mathbf{r})\cdot[\boldsymbol{\beta}_n(\mathbf{r}')\times\mathbf{n}(\mathbf{r})]dS, \tag{5}$$

$$I_2^{\mathrm{EFIE}}=\int_S\int \boldsymbol{\beta}_m(\mathbf{r})\cdot\left[\int_{S'}\int \boldsymbol{\beta}_n(\mathbf{r}')G(\mathbf{r},\mathbf{r}')dS'\right]dS, \tag{6}$$

$$I_3^{\mathrm{EFIE}}=\int_S\int \boldsymbol{\beta}_m(\mathbf{r})\cdot\nabla\left[\int_{S'}\int \nabla_S'\cdot\boldsymbol{\beta}_n(\mathbf{r}')G(\mathbf{r},\mathbf{r}')dS'\right]dS, \tag{7}$$

$$I_4^{\mathrm{EFIE}}=\int_S\int \boldsymbol{\beta}_m(\mathbf{r})\cdot\left[\int_{S'}\int \boldsymbol{\beta}_n(\mathbf{r}')\times\nabla G(\mathbf{r},\mathbf{r}')dS'\right]dS. \tag{8}$$

Similarly, for the MFIE part the following integrals have to be evaluated:

$$I_1^{\mathrm{MFIE}}=\int_S\int \boldsymbol{\beta}_m(\mathbf{r})\cdot\boldsymbol{\beta}_n(\mathbf{r}')ds, \tag{9}$$

$$I_2^{\mathrm{MFIE}}=\int_S\int \mathbf{n}(\mathbf{r})\times\boldsymbol{\beta}_m(\mathbf{r})\cdot\left[\int_{S'}\int \boldsymbol{\beta}_n(\mathbf{r}')G(\mathbf{r},\mathbf{r}')ds'\right]ds, \tag{10}$$

$$I_3^{\mathrm{MFIE}}=\int_S\int \mathbf{n}(\mathbf{r})\times\boldsymbol{\beta}_m(\mathbf{r})\cdot\nabla\left[\int_{S'}\int \nabla_S'\cdot\boldsymbol{\beta}_n(\mathbf{r}')G(\mathbf{r},\mathbf{r}')ds'\right]ds, \tag{11}$$

$$I_4^{\mathrm{MFIE}}=\int_S\int \mathbf{n}(\mathbf{r})\times\boldsymbol{\beta}_m(\mathbf{r})\cdot\left[\int_{S'}\int \boldsymbol{\beta}_n(\mathbf{r}')\times\nabla G(\mathbf{r},\mathbf{r}')ds'\right]ds. \tag{12}$$

The integrals I_1 for the EFIE and MFIE part are regular and can be evaluated numerically, using standard numerical integration routines without any special treatment. The integrals I_2 and I_3 for the EFIE and MFIE parts contain in the integrand the Green's function and singularities of order $1/R$ must be considered, when the observation point is near the source point. Finally, the integrals I_4 for the EFIE and MFIE parts contain in the integrand the gradient of the Green's function and singularities of order $\nabla 1/R$ must be considered. There are many methods that can be used to evaluate these integrals in the singular case, such as the Duffy's transformation and singularity extraction method.

3.1 Duffy's transformation

In this method the source triangle is first divided into three subtriangles with common vertex at singular point, as shown in Fig. 1. The inner source integral is then written in terms of these subtriangles as follows:

$$I=\int_S\int \frac{F(R,\mathbf{r},\mathbf{r}')}{R}ds=\sum_{e=1}^{3}\int_0^1\int_0^{1-\xi_1^e}\frac{F(\xi_1^e,\xi_2^e)}{R}J(\xi_1^e,\xi_2^e)d\xi_2^e d\xi_1^e, \tag{13}$$

where ξ_1^e and ξ_2^e are the parametric coordinates of the eth subtriangle and $J(\xi_1^e,\xi_2^e)$ is the corresponding Jacobian. The

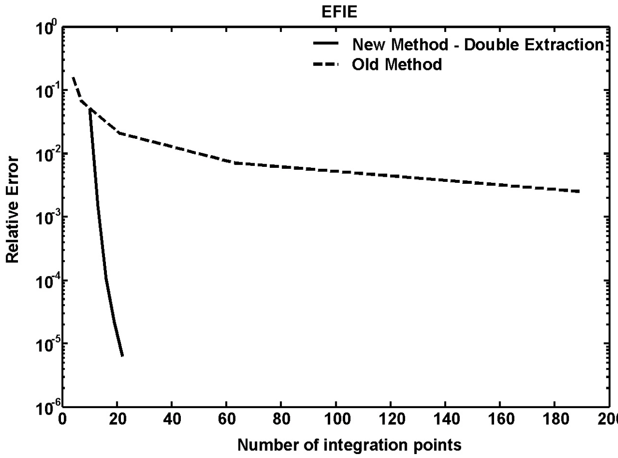

Fig. 5. Relative error of singular terms of EFIE coupling integrals with singularity $\nabla 1/R$ for the new method with double extraction and for the old method, normalized by the corresponding constant value after convergence.

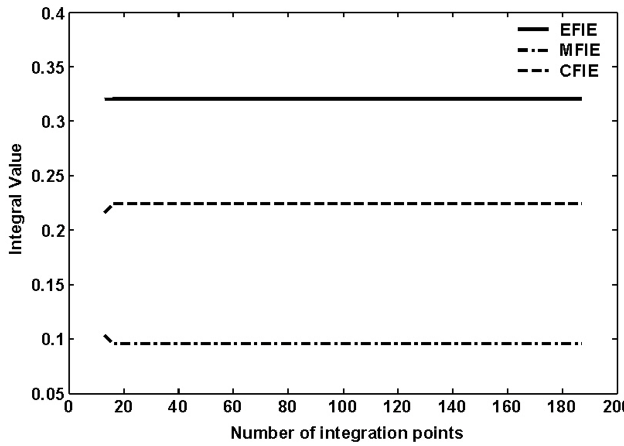

Fig. 6. Convergence of singular coupling integrals with singularity $\nabla 1/R$ for EFIE, MFIE and CFIE, using the new method with double singularity extraction.

integral over each subtriangle can be transformed into an integration over a square by introducing the transformation $\xi_2^e=(1-\xi_1^e)u$. By doing this, we have

$$I=\sum_{e=1}^{3}\int_0^1\int_0^1(1-\xi_1^e)\frac{F(\xi_1^e,(1-\xi_1^e)u)}{R}J(\xi_1^e,(1-\xi_1^e)u)d\xi_1^e du, \quad (14)$$

with

$$R=(1-\xi_1^e)\sqrt{q(\xi_1^e,u)}. \quad (15)$$

The function q is defined in terms of the coordinate ξ_1^e, the transformation variable u and the position vectors of the patch. The numerator $(1-\xi_1^e)$ cancels the singular nature of R.

3.2 Singularity extraction method

In this method, surface integrals are regularized by extracting a singular term from the Green's function as follows:

$$G(\mathbf{r},\mathbf{r}')=\left(G(\mathbf{r},\mathbf{r}')-\frac{1}{R}\right)+\frac{1}{R}. \quad (16)$$

Due to this, the gradient of the Green's function becomes

$$\nabla G(\mathbf{r},\mathbf{r}')=\nabla\left(G(\mathbf{r},\mathbf{r}')-\frac{1}{R}\right)+\nabla\frac{1}{R}. \quad (17)$$

The terms in parentheses of Eqs. (16) and (17) are regular and the extracted term includes the singularity. Applying this extraction to the integrals I_2-I_4 for the EFIE and MFIE parts, the inner source integral of the singular extracted term can be calculated in any case in closed form in primed coordinates using the formulas in (Wilton et al., 1984; Graglia, 1993; Eibert and Hansen, 1995). The remaining function is regular and the outer testing integral is being calculated numerically in unprimed coordinates. Further, the whole surface integrals I_2^{EFIE} and I_3^{EFIE} of the EFIE part can be calculated in closed form, if the test and source integrals coincide (Eibert and Hansen, 1995).

Difficulties in this approach may arise due to the following two reasons: First, after the extraction of the singular term from the Green's function, the remaining function in the parentheses of Eq. (16) has a discontinuous derivative at R=0. This discontinuity means that a straightforward application of a numerical integration routine for the testing integral may lead to an inaccurate solution. Second, difficulties may occur in the case of $\nabla 1/R$ (integrals I_4 for EFIE and MFIE part), when the source and test triangles have common points and are not in the same plane. In particular, although it is possible to calculate the inner source integral of the extracted singular term $\nabla 1/R$ analytically using the formulas in (Graglia, 1993), a logarithmic singularity still remains on the outer testing integral. Therefore, if higher accuracy is required, the testing integral cannot be calculated by a standard numerical calculation technique.

The above difficulties can be avoided by applying the singularity treatment recently proposed by Ylä-Oijala and Taskinen (Ylä-Oijala and Taskinen, 2003). In order to avoid the discontinuity of the remaining function in the parentheses of Eq. (16), an additional term can be extracted from the Green's function as follows:

$$G(\mathbf{r},\mathbf{r}')=\left(G(\mathbf{r},\mathbf{r}')-\frac{1}{R}+\frac{k^2}{2}R\right)+\frac{1}{R}-\frac{k^2}{2}R. \quad (18)$$

Due to this, the gradient of the Green's function becomes

$$\nabla G(\mathbf{r},\mathbf{r}')=\nabla\left(G(\mathbf{r},\mathbf{r}')-\frac{1}{r}+\frac{k^2}{2}R\right)+\nabla\frac{1}{R}-\frac{k^2}{2}\nabla R. \quad (19)$$

Now the terms in parentheses of Eqs. (18) and (19) have a continuous derivative at R=0 and calculation of source and testing integrals of this terms can be done easily with standard numerical procedure. The two extracted terms, i.e. the

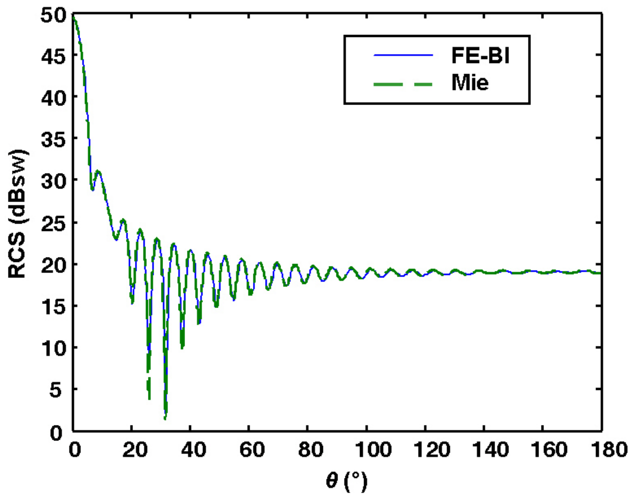

Fig. 7. Bistatic-RCS of thin-coated metallic sphere.

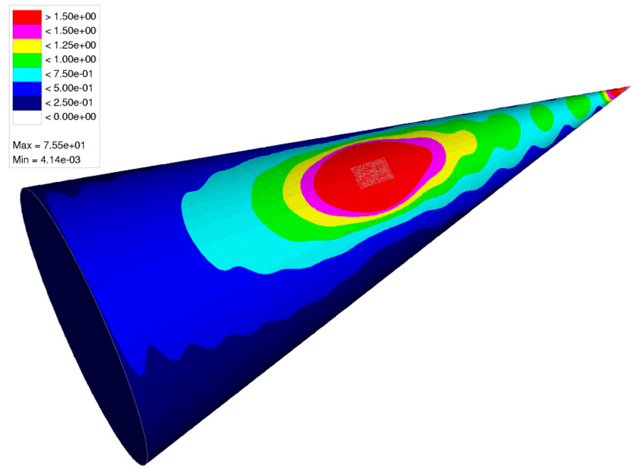

Fig. 8. Magnitude of the electric current density of a cavity-backed patch on a metallic cone.

second and third term of RHS of Eqs. (18) and (19), respectively can be integrated analytically over the source triangle in primed coordinates for all integrals I_2-I_4 using the formulas presented in (Graglia, 1993) and (Ylä-Oijala and Taskinen, 2003). The formulas for analytical integration presented in (Ylä-Oijala and Taskinen, 2003) are iterative and general, allowing extraction of any number of terms from the singular kernel and integration of these terms over the source triangle in closed form.

Thereafter, the last term of Eqs. (18) and (19) can be integrated numerically over the testing triangle in unprimed coordinates for all integrals I_2-I_4, because the outer integrand for this term is regular. The testing integral of the second term of Eq. (18) can be as well calculated numerically in unprimed coordinates for integrals I_2 and I_3 of the EFIE and MFIE part, because the outer integrand for this term is regular, too. The problem is to calculate the testing integral of the second term of Eq. (19), because in the outer integrand of this term a logarithmic singularity remains when the source and testing triangles have common points and are not at the same plane. This singularity exists in integrals I_4 of the EFIE and MFIE part. In particular, for analytical calculation of the source integral of the term $\nabla 1/R$ in primed coordinates the gradient is being divided into normal and surface components and the integral of the surface component over the source triangle is transformed with Gauss theorem into a line integral, which means that

$$\int\limits_{S'}\int \nabla\frac{1}{R}dS' = \int\limits_{S'}\int \nabla_n\frac{1}{R}dS' + \int\limits_{\partial S'} \mathbf{u}(\mathbf{r}')\frac{1}{R}dl'. \tag{20}$$

Now, according to Eqs. (30)-(32) in Graglia (1993) the analytical expression of the line integral in the above equation has a logarithmic term, which causes the logarithmic singularity in the outer testing integral. Note that the same logarithmic singularity exists as well in the integrals I_4 when applying the single extraction of Eq. (16). In order to avoid this singularity, according to (Ylä-Oijala and Taskinen, 2003), after the source integral of the surface component of the singular term $\nabla 1/R$ is transformed into a line integral, the order of integration is being changed.

For the EFIE part, the remaining term of integral I_4^{EFIE}, which produces the logarithmic singularity after applying the singularity extraction of either Eq. (17) or Eq. (19), has the form

$$I_4^{\mathrm{EFIE}} = \int\limits_S\int (\mathbf{r}-\mathbf{p})\cdot\left[\int\limits_{S'}\int (\mathbf{r}'-\mathbf{q})\times\nabla'\frac{1}{R}dS'\right]dS. \tag{21}$$

where $\mathbf{p}$ is the position vector of the free vertex of the testing triangle and $\mathbf{q}$ is the position vector of the free vertex of the source triangle. Note that in the above equation the identity $\nabla(1/R) = -\nabla'(1/R)$ has been used. Replacing $(\mathbf{r}'-\mathbf{q})$ by $(\mathbf{r}'-\mathbf{r})+(\mathbf{r}-\mathbf{p})+(\mathbf{p}-\mathbf{q})$ and separating the normal and surface derivatives, Eq. (21) can be written as

$$\begin{aligned} I_4^{\mathrm{EFIE}} = &\int\limits_S\int (\mathbf{r}-\mathbf{p})\cdot\left[(\mathbf{p}-\mathbf{q})\times\int\limits_{S'}\int \nabla_n'\frac{1}{R}dS'\right]dS \\ &+\int\limits_S\int (\mathbf{r}-\mathbf{p})\cdot\left[(\mathbf{p}-\mathbf{q})\times\int\limits_{S'}\int \nabla_S'\frac{1}{R}dS'\right]dS. \end{aligned} \tag{22}$$

The source integral of the normal derivative is calculated analytically in primed coordinates, using Eq. (26) in Graglia (1993). In the singular case, i.e. when the testing and source triangles are not in the same plane and they have common points, the surface gradient term is dominant and numerical integration of the normal component over the testing triangle can be done with reasonable accuracy. As said before, the analytical formula for the source integral of the surface gradient term in Graglia (1993) includes a logarithmic expression, which causes the logarithmic singularity when trying to evaluate the testing integral of the surface gradient term numerically. To avoid this, after using the Gauss theorem to translate the integral of the surface gradient over the

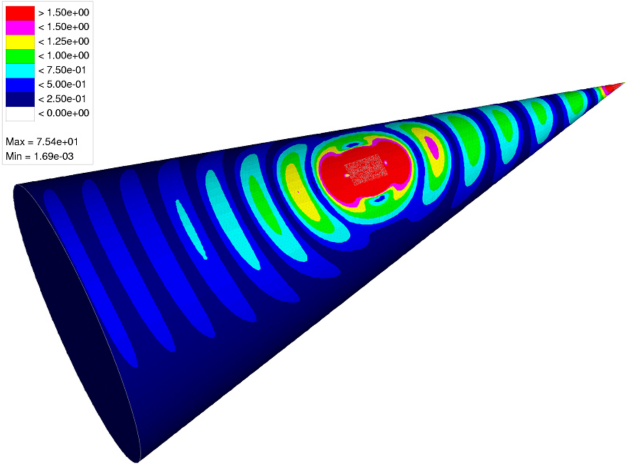

Fig. 9. Imaginary part of the electric current density of a cavity-backed patch on a metallic cone.

source triangle into a line integral, the order of integration is changed and the surface component of Eq. (22) can be written as

$$I_{4\text{surf}}^{\text{EFIE}} = \int_S \int (\mathbf{r}-\mathbf{p}) \cdot \left[(\mathbf{p}-\mathbf{q}) \times \int_{\partial S'} \mathbf{u}(\mathbf{r}') \frac{1}{R} dl' \right] dS$$

$$= \int_{\partial S'} \left[(\mathbf{p}-\mathbf{q}) \times \mathbf{u}(\mathbf{r}') \right] \cdot \left\{ \int_S \int \frac{(\mathbf{r}-\mathbf{p})}{R} dS \right\} dl'; \qquad (23)$$

where $\partial S'$ is the boundary of the source integral and $\mathbf{u}(\mathbf{r}')$ is the outer unit vector normal to $\partial S'$. Now, the inner testing integral is calculated analytically in unprimed coordinates using the formulas in (Ylä-Oijala and Taskinen, 2003). In those formulas the logarithmic term is cancelled. As a result, the outer source line integral has a regular integrand and allows numerical integration in primed coordinates. Hence, by considering the normal and surface gradients separately and changing the order of integration, the logarithmic singularity on the outer testing integral can be avoided.

The same procedure can be applied to the integral I_4^{MFIE} of the MFIE part. Here, the remaining term of integral I_4^{MFIE}, which produces the logarithmic singularity, after applying the singularity extraction of either Eq. (17) or Eq. (19), has the form

$$I_4^{\text{MFIE}} = \int_S \int [\mathbf{n}(\mathbf{r}) \times (\mathbf{r}-\mathbf{p})] \cdot \left[\int_{S'} \int (\mathbf{r}'-\mathbf{q}) \times \nabla' \frac{1}{R} ds' \right] dS. \qquad (24)$$

Applying the above procedure leads to following surface component of Eq. (24):

$$I_{4\text{surf}}^{\text{MFIE}} = \int_{\partial S'} \left[(\mathbf{p}-\mathbf{q}) \times \mathbf{u}(\mathbf{r}') \right] \cdot \left\{ \mathbf{n}(\mathbf{r}) \times \int_{\mathbf{S}} \int \frac{(\mathbf{r}-\mathbf{p})}{\mathbf{R}} \mathbf{dS} \right\} dl'$$

$$- \int_{\partial S'} \mathbf{u}(\mathbf{r}') \cdot \mathbf{n}(\mathbf{r}) \left\{ \int_{\mathbf{S}} \int \frac{|\mathbf{r}-\mathbf{p}|^2}{\mathbf{R}} \mathbf{dS} \right\} \mathbf{dl}'. \qquad (25)$$

Note that due to the cross product with the normal unit vector of surface S the last term of Eq. (25) must also be calculated. The inner testing integral of this term is calculated analytically in primed coordinates in (Ylä-Oijala and Taskinen, 2003) and the outer source line integral has a regular integrand and can be calculated numerically in primed coordinates.

In the next section, convergence results of the coupling integrals with singularity $\nabla 1/R$ are presented, using the above method within a hybrid FEBI technique with the CFIE. Further, numerical results of the bistatic-RCS of a thin-coated metallic sphere and of the surface currents of a cavity-backed patch on a metallic cone are presented.

4 Numerical results

The numerical treatment proposed by Ylä-Oijala and Taskinen has been applied in a hybrid FEBI technique, using the CFIE. The convergence of coupling integrals with singularity $\nabla 1/R$ has been calculated with this new method for various integral equations (EFIE, MFIE, CFIE) and for both the single extraction of Eq. (16) and the double extraction of Eq. (18). The computations have been made for the triangle combination (1, 2) of Fig. 2. These triangles lie perpendicular within a tetrahedron and have a common edge.

First, the value of the EFIE coupling integrals with singularity $\nabla 1/R$ given in Eq. (8) has been calculated with the new method for single and double singularity extraction and for various integration points. The convergence of this integration is compared with the corresponding values using the old method, where only one term is extracted and integrated analytically over the source triangle and Gaussian quadrature is applied for the remaining outer testing integral. The results of this comparison are shown in Fig. 3 for the singular terms and in Fig. 4 for the regular terms of the coupling integrals. Note that the value of the regular terms of the coupling integrals with the old method is the same with the corresponding value with the new method and single extraction, because in both cases singularity was extracted by the same way and surface integrals were calculated numerically with the same routine.

It can be noticed that the coupling integrals converge faster with the new method for both singular and regular terms. Especially for the singular terms, after only a few integration points the coupling integrals using the new method have reached a constant value for single extraction, as well as a constant value for double extraction. These constant values are different because of the additional extracted term. Note that the integral value with the old method converges to the constant value of the new method with single extraction, because in both cases the same singularity extraction has been used.

The two constant values (for single and double extraction) after convergence can be used as a reference for the relative error of the value of the coupling integrals with the old method and with the new method and double extraction respectively, as shown in Fig. 5. It can be seen that the relative error for both methods becomes less by increasing the integration points, but with the old method this error still has a significant value after a lot of integration points. With the new method the relative error vanishes after only a few integrations points.

Next, convergence of the singular terms of the coupling integrals with singularity $\nabla 1/R$ has been compared for EFIE, MFIE and CFIE (with α=0.5), using the new method with double singularity extraction. The results of this comparison are shown in Fig. 6. It can be seen that for all integral equations using the new method the coupling integrals converge very fast and with about the same number of integration points.

Finally, the calculated bistatic-RCS of a thin-coated metallic sphere using the new singularity treatment within a hybrid FEBI technique with CFIE is shown in Fig. 7. The metallic kernel has a diameter of $D=10\lambda$ and the lossy thin layer a dielectric constant of $\varepsilon_r=2.5-0.05\,\mathrm{j}$. The magnitude of the electric current density of a cavity-backed patch on a metallic cone for $f=1.105\,\mathrm{GHz}$ is shown in Fig. 8. The appropriate imaginary part of the current density is shown in Fig. 9.

5 Conclusions

In this work, several techniques for the treatment of singular integrals for MoM solutions of surface integral equations have been reviewed. Especially for the integrals with singularity $\nabla 1/R$ a recently proposed method by Ylä-Oijala and Taskinen has been considered in detail, because of its capability to avoid effectively logarithmic singularity for those integrals. This method has been evaluated within a hybrid FEBI technique using CFIE where also higher-order singularity extraction was applied. The advantages of this method have been shown in convergence results for the singular case, where two triangles are not at the same plane and have common points. It was found that the coupling integrals with singularity $\nabla 1/R$ converge faster using the new method. Further, it can be noticed that the higher-order extracted terms from the singular kernel additionally regularize the coupling integrals. Because of this, accuracy, robustness and computation time of the FEBI technique is improved.

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