Systematic methods for the synthesis of equidistant MIMO arrays

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Abstract. Finding a feasible antenna arrangement for multiple input multiple output (MIMO) arrays to serve a specific purpose is a first crucial step towards a successful MIMO radar system design. Design methods to synthesize uniformly weighted and equidistant MIMO arrays are proposed and investigated. The methods can be used to gain a design foundation for 1D or 2D arrays without software tools or programming effort. Since the presented approach does not consider electromagnetic fields, electromagnetic full-wave simulations might be required additionally. The method is based on sequentially copying and displacing antenna groups with the help of a number scheme. A nomenclature is proposed to classify the degrees of freedom in the design procedure. If the antennas are aligned to a uniform grid, a polynomial representation of the array can be chosen alternatively. This method is beneficial when redundancies of a produced array and where they appear must be analyzed. A new design problem arises when an array is to consist of only transceiving antennas, which can be analyzed with polynomial multiplication. One strategy to find a suitable MIMO array consisting of transceiver elements is given and evaluated.

1 Introduction

Using a multiple input multiple output (MIMO) array in radar applications can significantly reduce the number of required physical antennas of a radar system. This reduces the system complexity and can have economic advantages. In the radar domain, MIMO arrays are used in several applications and fields. Feger et al. (2009) use a non-uniform prototype MIMO array for far field 2D imaging and show potential applicability of their design in automotive scenarios. Other examples are public security scanning (e.g., Gao et al., 2018; Zhuge and Yarovoy, 2011) or package control (Yanik et al., 2020), where the penetrability of materials like textiles or cardboard is exploited. By using individual transmitter (Tx) and receiver (Rx) channels, each combination of them yields a full radar return signal. Therefore, the number of required physical antennas can be reduced. From a model point of view, a Tx-Rx pair can be modeled as a virtual element (VE). All antenna pairs of each combination result in a virtual array (VA). To analyze a structure of Tx and Rx antennas, the method of convolution and scaling is commonly used (Ender and Klare, 2009). However, for the synthesis of a VA, no straightforward method exists. Generic non-uniform MIMO arrays are useful to compromise resolution, sidelobe-levels and element-density in a controllable way. A suitable non-uniform array can be found by discrete optimization algorithms like particle swarm optimization (Schmid et al., 2009) or genetic algorithms (Huang et al., 2021). The signal processing for non-uniform arrays is computationally more complex in general, since no geometric structure can be exploited. On the other hand, efficient FFT-based methods can be used for uniform arrays, e.g. as given by Sheen et al. (2001) for 3D imaging. A specific type of MIMO arrays are the ones having equidistant virtual elements without overlapping items or gaps. When using uniform MIMO arrays for near-field imaging, signal corrections might be necessary to achieve the desired reconstruction accuracy (Yanik and Torlak, 2019). For a radar signal processing algorithm that fully relies on the virtual antenna concept, these overlapping VEs would deliver redundant information and the physical resources would not be exploited optimally in the sense of amount of elements. In this paper, a design method for equidistant MIMO arrays is introduced.
different locations. If the distance
In a bistatic radar scenario, Tx and Rx are distinct antennas at
array are explained.
In this section, the concepts of a virtual antenna and a virtual
elements. Finally, Sect. 6 concludes the paper and gives an
outlook on future work.

The remainder of the paper is structured as follows: Sect. 2
explains the concept of a virtual array and the method of con-
volution and scaling. Section 3 introduces the proposed de-
sign method for equidistant MIMO arrays. In Sect. 4, an ex-
ample design for a 2D MIMO array is shown. Section 5 ex-
plains the polynomial representation and introduces an itera-
tive method for TRx antennas is described and rated in different
aspects.

The main focus of this paper is to introduce a simple design
method for an equidistant and equally weighted VA along x. Such
a VA has the form

\[ F_{VA}(x, y_{VA}) = \sum_{i=0}^{L-1} \delta(x - k \cdot i, y_{VA}) \]  

(3)

with size L and a scaling factor k. The overall length of the
VA is k(L−1). The VA forms a grid along x at y_{VA}. Note, that
each element in the sum has a weighting of 1, which means
exactly one Tx-Rx pair contributes to this VE location. To
obtain functions \( F_{Tx} \) and \( F_{Rx} \), a straightforward method is
explained in the following sections.

3 Design method for one-dimensional equidistant
arrays
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3.1 Establishment of requirements
In order to generate a regular MIMO array in the required
form of Eq. (3) with size L, the prime factorization of L is
done first. The P prime factors \( p_i \in \mathbb{N} \) with the property

\[ L = \prod_{i=0}^{P-1} p_i \]  

(4)

themselves are of course primes and cannot be divided fur-
ther. It is recommended to choose L, such that the prime fac-
tors \( p_i \) will not get too large. This adds more design flexi-
bility in the method.
3.2 Theoretical outline

From this point onwards, a mixed basis number system will be introduced. Unlike the well-known decimal, binary or hexadecimal number systems, each digit is assigned its own basis. The number system will be represented by a \( W \)-element sequence of numbers \((a_0, a_1, \ldots, a_{W-1})\), where \((\langle \rangle)\) denotes an ordered, finite sequence of numbers and \(a_i\) the base of the digit at position \(i\), e.g., a three-digit number with basis \((4, 3, 2)\) will have a binary least significant digit. The most significant digit has base 4 and the one in between is ternary. Counting in this number system would yield 000, 001, 010, 011, 020, …, 320, 321. As seen, the amount of displayable numbers in such a number system is finite. A number \(y\) in a mixed basis system can be represented as a concatenation of \(W\) symbols in the form

\[
y = d_0d_1\ldots d_{W-1} \in S, \quad d_i \leq a_i - 1,
\]

where \(S\) denotes the set of all displayable numbers in a number system \(S\). The conversion \(\text{conv}(y; S)\) of an integer \(y\) from a number system \(S\) into a decimal \(z\) is calculated as

\[
z = \text{conv}(y; S) = d_{W-1} + \sum_{i=0}^{W-2} d_i \cdot \left( \prod_{j=i+1}^{W-1} a_j \right).
\]

Next, each prime factor is either assigned a subscript \(t\) for Tx or \(r\) for Rx. This can be notated as

\[
(p_i \leftarrow p_{i,t}) \oplus (p_i \leftarrow p_{i,r}) \quad \forall i \in \mathbb{N} < W,
\]

where \(\oplus\) denotes a logical exclusive or operation. The sequence \(P\) is defined as an ordered list of the annotated prime factors in the form

\[
P = \langle p_0, [t|r], p_1, [t|r], \ldots, p_{P-1}, [t|r] \rangle.
\]

\([t|r]\) denotes, that either the subscript \(t\) or \(r\) is chosen for each element, but not both. Note, that the choice of a particular assignment as well as the order of the prime factors in \(P\) will influence the inner structure of the resulting VA and should be chosen in such a way as to design the most feasible MIMO array. A particular number system \(S\) with the ordered prime factors from Eq. (8) is now considered. It is in the form

\[
S = \langle p_0, p_1, \ldots, p_{P-1} \rangle
\]

with \(P = W\). For the Tx and Rx antennas, sets of enumerated numbers \(T_k\) and \(R_k\) in the number system of Eq. (9) are created respectively. Therefore, the assignment of subscripts \(t\) and \(r\) from Eq. (8) is used. For \(T_k\) all possible numbers in the number system are listed, where the digits at locations with \(p_{i,t}\) are zero. For \(R_k\), respectively, all possible numbers are listed of which the digits \(d_i\) are zero, if \(p_i \leftarrow p_{i,t}\). Formally, this can be written as

\[
T_k = \{ x \in \mathbb{S}_S | p_i \leftarrow p_{i,t} \Rightarrow d_i = 0 \}.
\]

and

\[
R_k = \{ x \in \mathbb{S}_S | p_i \leftarrow p_{i,t} \Rightarrow d_i = 0 \}.
\]

The number of elements in \(T_k\) is given by

\[
|T_k| = \prod_{i=0}^{L-1} p_i
\]

and in \(R_k\) respectively

\[
|R_k| = \prod_{i=0}^{L-1} p_i.
\]

Each number in \(T_k\) is used as a position coordinate for one transmitter antenna in the MIMO design. In the same way, the elements of \(R_k\) are used as coordinates for the receiving antennas. A MIMO system can have at most one distinct VE location for each Tx-Rx pair. Hence, the number of VEs is limited to \(|T_k| \cdot |R_k| = N_{\text{Tx}} \cdot N_{\text{Rx}} = L\) in the design method introduced here. Therefore, the separation into Tx and Rx antennas is feasible, as it is capable of producing a MIMO array according to Eq. (3).

All physical antennas lie on a line parallel to the \(x\)-axis at \(y_{\text{Tx}}\) for Tx and \(y_{\text{Rx}}\) for Rx respectively. Consequently, every element of the VA is located along \(y\) at

\[
y_{\text{VA}} = \frac{y_{\text{Tx}} + y_{\text{Rx}}}{2}.
\]

The same applies to the \(x\)-coordinate for each possible Tx-Rx pair. The spatial scaling of \(\frac{1}{2}\) will be irrelevant for the following argument and will hence be disregarded. Each element in \(T_k\) and \(R_k\) determines the \(x\)-coordinate of one antenna according to

\[
x = 2k \cdot \text{conv}(d; S),
\]

where \(k\) denotes the scaling factor from Eq. (3). The VA returned from the Tx and Rx positions, which are now fully determined, satisfies the required form from Eq. (3). This can be seen, when the sum of the \(x\)-components of Rx and Tx is performed in the domain of number system \(S\). Any element \(a\) in \(T_k\) with symbols \(t_i\) added to any element \(b\) in \(R_k\) with symbols \(r_i\) yields a decimal result

\[
\text{conv}(a; S) + \text{conv}(b; S) = t_{W-1} + \sum_{i=0}^{W-2} t_i \cdot \left( \prod_{j=i+1}^{W-1} a_j \right) + r_{W-1} + \sum_{i=0}^{W-2} r_i \cdot \left( \prod_{j=i+1}^{W-1} a_j \right) = \langle[t|r]w-1 + \sum_{i=0}^{W-2} [t|r]i, \prod_{j=i+1}^{W-1} a_j \rangle.
\]
The elements of $T_S$ and $R_S$ for the first example with $P_1 = \langle 2r, 3t, 5r, 2t \rangle$.

<table>
<thead>
<tr>
<th>$T_S$ (in $S$)</th>
<th>$T_S$ (decimal)</th>
<th>$R_S$ (in $S$)</th>
<th>$R_S$ (decimal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>0</td>
<td>0000</td>
<td>0</td>
</tr>
<tr>
<td>0001</td>
<td>1</td>
<td>0010</td>
<td>2</td>
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<tr>
<td>0100</td>
<td>10</td>
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<td>0101</td>
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<td>0030</td>
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<td>0200</td>
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<tr>
<td>0201</td>
<td>21</td>
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<td></td>
<td>1010</td>
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<td>1040</td>
<td>38</td>
<td></td>
</tr>
</tbody>
</table>

Figures 2 and 3 show the generated distributions of antennas and their VA for the two exemplary assignments of $P$. The first distribution, derived from $P_1$ in Eq. (17), has six Tx antennas divided into three groups of two. The reason for this structure are the elements $2t$ and $3r$ in $P_1$. The second example has 12 Tx antennas and reveals a completely different structure in Fig. 3. The choice of $P$ influences the balance between the number of Tx and Rx and gives the designer some freedom about structuring the antennas into groups.

This simplification can be done due to the way, $T_S$ and $R_S$ are defined. For each digit, either the symbol $t_i$ or $r_i$ is zero. When computing the sum in $S$, no carry events are happening and the enumeration, each number in $S$ is reached exactly once through the combination of each one element from $T_S$ and $R_S$. Therefore, the set of all Tx-Rx combinations yields $S_S$. It follows that a VE for a particular position along the $x$-axis is generated from exactly one Tx-Rx pair.

### 3.3 Design examples

To illustrate the working principle of this design concept, two different examples are shown that generate a VA with size $L = 60$. More examples and a more small-step explanation of the method can be found in Holder and Eberspächer (2022). The prime factors $\{2, 3, 5\}$ of $L$ can be ordered and assigned with the subscripts $t$ or $r$ arbitrarily. For the first run,

$$P_1 = \langle 2r, 3t, 5r, 2t \rangle$$

is chosen. The used number system is $S_1 = \langle 2, 3, 5, 2 \rangle$. The elements of $T_S$ and $R_S$ are given in $S$ and as decimals in Table 1.

As a second possible assignment,

$$P_2 = \langle 2t, 5r, 2r, 3t \rangle$$

is chosen. The used number system here is $S_2 = \langle 2, 5, 2, 3 \rangle$. The elements of $T_S$ and $R_S$ for this second example are given in Table 2.

### 3.4 Nomenclature

In the previous sections, a design method for equidistant and equally weighted MIMO arrays was introduced. The designer has two degrees of freedom when deciding for a sequence $P$ of subscripted prime factors: The order of the prime factors and their individual assignment to Tx ($t$) or Rx ($r$). To classify this design freedom, a nomenclature is pro-
posed. The chain of characters

\[ [T|R]_{x_0}[T|R]_{x_1}\cdots[T|R]_{p-1} \quad (19) \]

uniquely identifies a MIMO array. The letter \( R \) or \( T \) determines, whether this prime factor is assigned to \( r \) or \( t \). The subscripts \( x_i \) give the number system \( S \). Adjacent digits with the same assignment can be accumulated in one digit with a base corresponding to the product of the individual bases, e.g., the subsequence \( R_3R_2 \) will result in the same virtual array as \( R_2R_3 \) or \( R_6 \). To avoid this ambiguity, each basis should be accumulated such that \( T \) and \( R \) appear in an alternating fashion. So, the array descriptor \( T_2T_2R_3R_3T_3 \) should be converted to \( T_4R_1T_3 \) to increase readability. In favor of generality, arrays with only one \( Tx \) or one \( Rx \) can be notated as \( T_sR_1 \) or \( R_sT_1 \) respectively. Although strictly speaking they are not MIMO arrays.

### 3.5 Shifting lemma

Independent of the designed array, shifting all \( Rx \) by a spatial vector \( s_{Rx} \) or all \( Tx \) by \( s_{Tx} \) does not affect the structure of the VA. \( Tx \) and \( Rx \) positions (encoded as a vector \( p = (x, y)^T \) here)

\[ p_{Rx,i} = p_{Rx,i} + s_{Rx}; \quad p'_{Tx,j} = p_{Tx,j} + s_{Tx} \quad (20) \]

are each shifted by an offset vector. The VA itself is just displaced by the arithmetic mean of \( s_{Tx} \) and \( s_{Rx} \) since the resulting position

\[ p'_{VA,i,j} = p_{VA,i,j} + \frac{s_{Tx} + s_{Rx}}{2}, \quad (21) \]

of the VE is shifted by the same amount for each \( i < N_{Rx} \) and \( j < N_{Tx} \).

### 4 Example design for a two-dimensional array

The concept of continuously building up antenna groups through digit assignment can also be implemented for the case of a 2D MIMO. In order to create a regular MIMO array in two dimensions, one could either make use of a square grid or a hexagonal grid. A comparison is given by Wagner et al. (2018). Dahl et al. (2017) investigated fractal design approaches on a hexagonal grid, which as well are structured methods to design MIMO arrays of arbitrary size. In this paper, a square grid is used to enable the use of fourier based 3D imaging algorithms (Sheen et al., 2001). For this, the method introduced in the previous section is applied to the vertical and horizontal direction separately. A more practical example is shown using a 80 GHz radar chip.

The goal is to create a 2D MIMO array with the AWR2243 radar transceiver IC from Texas Instruments Incorporated (2020). The chip operates from 76 GHz up to 81 GHz using the FMCW principle. It has three \( Tx \) and four \( Rx \) channels suitable for MIMO operation. Additionally, multiple chips can be coherently cascaded to operate with an even larger number of antennas. In this example, three chips are used to create an effective size \( 9 \times 12 \) MIMO array. To get an array size of \( L_x = 12 \) and \( L_y = 9 \), the pattern \( R_2R_3R_2 \) is used in horizontal and \( T_3R_3 \) in vertical direction. As the produced MIMO array is intended for direction of arrival (DOA) estimation (Chen et al., 2010), a maximum scaling factor of \( k \approx \lambda/4 \approx 3.8 \text{ mm} \) (at center frequency) is allowed theoretically, in order to avoid aliasing. As shown by Zhuge and Yarovoy (2011), this constraint can be relaxed for radar systems with very high fractional bandwidth. The scaling factor \( k \approx 2 \text{ mm} \) is defined through Eq. (15) since the available bandwidth is low. The result is shown in Fig. 4. The shifting property from Eq. (21) is used obtain a symmetric design with a common centroid layout. The next step would be to design a patch antenna and to place it at the positions for \( Rx \) and \( Tx \). The microstrip routing to the three chips should then be convenient, as the \( Tx \) antennas are already clustered in groups of three.

A drawback of this placement choice is that \( Rx \) and \( Tx \) antennas are close together in the middle row. Populated with patch antennas, this small physical distance would lead to unwanted crosstalk. Alternatively, \( Tx \) and \( Rx \) could be separated on the circuit board as shown in Fig. 5 by using the shifting property. This leads to larger overall space requirements, but helps reducing crosstalk. Another solution could be to increase the scaling factor \( k \), if the application can implicitly rule out ambiguities from aliasing by design.

![Figure 4. 2D MIMO array design example. Tx positions (blue squares), Rx positions (black triangles) and resulting VA (green circles).](https://doi.org/10.5194/ars-21-15-2023)
4.1 Analysis of published arrays

Numerous other publications already proposed MIMO arrays, which can be created with this method. Zhuge and Yarovoy (2012) are using a 2D array \(R_2T_3R_2\) in both vertical and horizontal direction) as a reference for sparse MIMO near-field imaging investigations. Ender and Klare (2009) use an ARTINO type MIMO array in the form \(T_2R_{16}T_{16}\), which is installed along the wings of an aeroplane. Another example is a slightly stretched version of a \(T_2R_{16}T_8\) array (Herschel et al., 2016) for a passenger security system.

5 The usage of transceiver antennas

This section describes the special case when TRx antennas are used instead of distinct Tx and Rx. First, an alternative view to convolution and scaling, namely polynomial multiplication is introduced.

5.1 Polynomial multiplication

The concept of polynomial multiplication is not only applicable to TRx antennas, but also distinct Tx and Rx, as in Sect. 3. If Tx and Rx antennas are located on a uniform grid with a defined zero-location, the VA can also be calculated via polynomial multiplication. The set of Tx antennas is therefore expressed as a polynomial \(t(x)\). A coefficient \(c_i\) is either 1, if a transmitter antenna is present at this grid point or 0, if not. From the example in Fig. 2, \(t(x) = 1 + x + x^{10} + x^{11} + x^{20} + x^{21}\) would be returned, while \(r(x) = 1 + x^2 + x^4 + x^6 + x^8 + x^{30} + x^{32} + x^{34} + x^{36} + x^{38}\) would describe the Rx positions.

The occurring exponents show where Tx antennas are located. The same applies to Rx yielding a second polynomial \(r(x)\) with binary coefficients \(d_j\). The VA is now computed via polynomial multiplication as

\[
u(x) = t(x) \cdot r(x) = \sum_{i=0}^{\deg(t(x))} \sum_{j=0}^{\deg(r(x))} c_i d_j x^{i+j},\]

where \(\deg(\cdot)\) denotes the degree of a polynomial. The exponent of a term in \(\nu(x)\) indicates the position of the VE while the corresponding coefficient shows, how many Tx-Rx pairs contribute to it. As for the convolution method, the VA has to be spatially scaled by \(\frac{1}{2}\). However, the scaling is not relevant for analyzing the structure of the VA.

5.2 Structured approach for TRx MIMO arrays

If instead of dedicated transmitters and receivers, transceivers are used, the initially proposed methodology still works but leads to more redundancy. When each antenna is used as Tx and Rx (TRx), there is no difference between Tx and Rx anymore. This can be understood with the thought, that the sets \(T_x\) and \(R_x\) now consist of the set unions

\[
T'_x = R'_x = T_x \cup R_x
\]

of the initial sets.

The obtained VA from a given regular antenna layout can be represented as the result of a polynomial multiplication. Since the antennas are indistinguishable, \(t(x) = r(x) = a(x)\) holds, where \(a(x)\) denotes a polynomial encoding of the TRx positions. Given that the VA is now calculated as

\[
u(x) = (a(x))^2.
\]

It can be seen from any TRx structure with \(N_{TRx} \geq 2\) antennas, that redundancy is now implicitly present. The VA originated from two TRx antennas has the form

\[
u(x) = \left(1 + x^k\right)^2 = 1 + 2x^k + x^{2k}
\]

for an arbitrary positive integer \(k\). The value 2 in the cross-term coefficient shows, that this structure already contains redundancy as two VEs overlay each other at this position.

Let a TRx array with polynomial \(a(x)\) have degree \(n\). This means the rightmost TRx element is at position \(n\) and the highest nonzero term of \(a(x)\) is \(x^n\). Assume that the resulting VA does not have any gaps, i.e. \(v_i \geq 1\) for \(i \in [0, 2n]\) holds for the coefficients \(v_i\) of \(\nu(x)\). Now consider an extended TRx array, which is composed of \(a(x)\) and a shifted copy of \(a(x)\). It has the form \(a'(x) = a(x) + x^{2n+1} \cdot a(x)\). The resulting extended VA has the form

\[
u_e(x) = (a_e(x))^2 = a(x)^2 \cdot \left(1 + 2x^{2n+1} + x^{4n+2}\right).
\]
Figure 6. Coefficients of the polynomials \( a(x) = 1 + x \) and \( v(x) = 1 + 2x + x^2 \) for \( N_{\text{iter}} = 1 \) in Algorithm 1.

Since \( a(x)^2 \) covers all VE positions from zero up to \( 2n \), the second and third term fully cover the positions ranging from \( 2n + 1 \) to \( 4n + 1 \) and from \( 4n + 2 \) to \( 6n + 2 \), respectively. By copying and shifting the existing structure, the VA can be extended to three times the original length. This step can be cascaded several times to build up a large VA. The following iterative procedure illustrates this: The first iteration starts with a single TRx antenna represented by a polynomial \( a_0(x) = 1 \) of degree zero. Algorithm 1 shows the procedure to get the TRx MIMO array.

Algorithm 1 Iterative method to design a TRx MIMO array.

\[
\begin{align*}
a(x) &\leftarrow 1 \\
n &\leftarrow 0 \\
\text{for } i \text{ from 1 to } N_{\text{iter}} \text{ do} & \\
a_e(x) &= a(x) + x^{2n+1}a(x) \\
a(x) &= a_e(x) \\
n &= 3n + 1 \\
\text{end for}
\end{align*}
\]

Figures 6 and 7 show the results for \( N_{\text{iter}} = 1 \) and \( N_{\text{iter}} = 2 \) of Algorithm 1. Note that the coefficients of \( a(x) \) are shown at stretched positions to visualize the scaling effect, when the VA is generated. Figures 8 and 9 show the coefficients for \( N_{\text{iter}} = 3 \) and \( N_{\text{iter}} = 4 \), respectively. It can be observed that the coefficients get larger over the iterations. While the number of TRx doubles, the MIMO array length is multiplied by three for each iteration.

Compared to the number approach of the previous sections, there are fundamental differences: The redundancy for TRx arrays does increase with the \( N_{\text{iter}} \), whereas the number approach was designed to not permit redundancy at all. In exchange, the TRx arrays rely on strictly repeating structures, which lowers the hardware complexity. If the spacing is not feasible inside one iteration, one could always use a smaller shift between the two subarrays to get \( a_e(x) \). For the iteration, the alternatively produced \( a(x) \) from the previous iteration could be used without problems. Additionally, the physical size of the VA coincides with the extent of TRx antennas. For the number system approach, this is never the case.

For a selection of \( N \) antennas, the theoretical maximum size \( L \) for a regular VA without gaps is \( \frac{N(N+1)}{2} \) for TRx elements. If the antennas are distinct transmitters or receivers, it is \( \frac{N^2}{2} \). So roughly half of the size is achieved compared to TRx for large \( N \). While the number system approach reaches

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5.3 Design example

One significant benefit of the approach followed is the resulting modularity of the system. To demonstrate this advantage, a conceived design example is presented in this section. Consider a modular system design, where 16 TRx are used in four modules with four antennas per module. They are operating at a center frequency of 200 GHz. To achieve a sampling density of \( \lambda/2 \) of the VA at the center frequency, the minimum distance between two TRx is set to 1.5 mm. The transmitter of each TRx can be deactivated to be operated as a receiver only. With only one active transmitter at a time, TDM could be easily realized. Figure 10 shows an outline of such a submodule. It has four TRx antennas arranged in a grid of five with a void element in the middle. Note that this is exactly the obtained structure for \( N_{\text{iter}} = 1 \) in Algorithm 1, shown in Fig. 7.

These modules are now placed onto a carrier frame with an arrangement as shown in Fig. 11. First (corresponding to iteration 3 in Algorithm 1) the second module is placed with four units of free space next to the leftmost one. This structure is copied again at the right side. Note, that the free space in the middle is only 10 units wide. However, according to Algorithm 1, it should be 13. It was reduced here to meet an overall maximum length constraint of 60 mm. The resulting VA is therefore reduced in size from 81 to 75. Being able to reduce the shifting distance during one iteration is another benefit of this approach, which increases the flexibility during the design process.

6 Conclusions

In this paper, two design methods for creating uniform and equidistant MIMO arrays for 1D and 2D applications were introduced. A nomenclature to classify the degrees of freedom in the first method was proposed. Based on a design example for a 2D MIMO array, the versatility of this method was demonstrated. This was underlined by already published designs of other authors, which all could be created with the introduced design method. As a special case, a design method with TRx antennas was introduced and compared. It was shown that redundant virtual elements are implicitly present in this case.

On the other hand, the examples also highlighted limitations of the methods. Design goals and constraints like the physical size of an antenna (and therefore the minimum spacing) or routing constraints are not implicitly considered. Moreover, the special interest of the authors lies in the application of a MIMO structure for near-field imaging. Zhuge and Yarovoy (2011) point out that array factors are position dependant and more complex to determine in a near field scenario, which will be considered in future work. Removing the constraint of equal weighting in the VA, one could either choose to design redundant arrays \( (L < N_{VE}) \), sparse arrays \( (L > N_{VE}) \) or a combination of both. Both options will be considered in the near future, investigating the effect of an array choice on the realisability and performance of the radar. Especially, the use of a non-convex optimization algorithm lies in the interest of the authors. Another aim of the authors is to find a more elegant and flexible way to design TRx MIMO arrays. A non-trivial two-dimensional extension of the introduced TRx method must be found as well.
Data availability. No data sets were used in this article.

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