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# Evolution of Electromagnetics in the 19th Century

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**Abstract.** Steps leading to the present-day electromagnetic theory made in the 19th Century are briefly reviewed. The progress can be roughly divided in two branches which are called Continental and British Electromagnetics. The former was based on Newton's action-at-a-distance principle and French mathematics while the latter grew from Faraday's contact-action principle, the concept of field lines and physical analogies. Maxwell's field theory and its experimental verification marked the last stage in the process.[1]

## 1 Introduction

### 1.1 18th Century

The first serious mathematical theory for electromagnetics was constructed by the German Franz Ulrich Theodosius Aepinus (1721–1802) in his treatise (Aepinus, 1759). He considered attractive and repulsive electric and magnetic forces similar to Newton's action-at-a-distance gravity force with no medium effect. Force law was taken in the general form $1/r^n$ with an unspecified value for $n$. However, qualitative properties could be derived without actually knowing the exact value of $n$. Quantitative conclusions required the definite value $n=2$, which was determined after the famous measurements in 1785 by Charles-Augustin Coulomb (1736–1806) for both the electric and the magnetic force. Coulomb also showed experimentally that the electric force at the surface of a metal object is proportional to what he considered as the thickness of the electric fluid confined to the surface. He also held a view that electricity and magnetism are two separate phenomena with no coupling between them, which probably slowed down the interest to find any connection between them.

### 1.2 Galvanism

After Alessandro Volta (1745–1827) had introduced in 1800 a chemical source of electricity, the Volta pile, William Hyde Wollaston (1766–1828) compared the electricity obtained from it (galvanic electricity) to static electricity as less intense but being produced in much larger quantity. The vague concepts "intensity" and "quantity" of electricity stayed in use for three decades before the more exact concepts "voltage" and "current" were defined after the work of Georg Simon Ohm (1787–1854). In particular, the electric quantity (current) was not a clear concept because of competing single-fluid and two-fluid theories of electricity. Electric current in a conductor could be considered as due to movement of a single fluid in one direction or symmetric movement of two fluids in the opposite directions. Two-way movement was often favoured because it seemed to explain the reactions at the two electrodes in an electrolytic solution.

### 1.3 Electromagnetism

The connection between electricity and magnetism was first reliably shown by Hans-Christian Oersted (1777–1851) in 1820. In his interpretation, closing a circuit containing a Volta pile caused a "conflict of electricity" inside and outside the connecting wire, which was capable of deflecting the magnetic needle. This effect gave the possibility for detecting the electric quantity by using a magnetic needle, which later developed to the galvanometer. Oersted correctly described the strange nature of magnetic force around the current conductor (right-hand rule). He also showed that a magnet exerts a force on a current loop. After many unsuccessful attempts the converse phenomenon, the electromagnetic induction, was found experimentally in 1831 by Michael Faraday (1791–1867) and Joseph Henry (1797–1878). This led to the development of practical electromagnetic generators producing cheap electric energy and, finally, caused the electrification of the globe.

## 2 Continental electromagnetics

### 2.1 Electrostatics and magnetostatics

Coulomb's force law made electricity and magnetism a part of exact sciences. Being similar to Newton's action-at-a-distance law, it inspired a surge of mathematical research. Since this was based on best French traditions and done mainly on the European Continent, it can be called as Continental electromagnetics. Simon-Denis Poisson (1781–1840)

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[1]This article is a summary of the complete text which will appear in Lindell (2005).

gave in 1812 the first serious mathematical theory of electrostatics by determining the thickness of the electric fluid on a conducting ellipsoid and on two separated conducting spheres by requiring vanishing of the electric force inside conducting objects. He extended Laplace's partial differential equation from gravitational to electrostatic potential. In 1824 Poisson published a corresponding theory for magnetostatics. He defined polarization of magnetic material, the magnetic moment and equivalent magnetic surface charges. Poisson's theory was generalized by George Green (1793–1841) in a privately published article in 1828, which introduced the Green function and Green's theorem. The article, however, went unnoticed during his lifetime.

### 2.2 Ampère's force law

After Oersted's discovery, André-Marie Ampère (1775–1836) demonstrated in 1820 that there is a force between electric currents. In 1826 he formulated the basic law for the force between two differential current elements (Ampère, 1826). The law obeyed Newton's principle of action and reaction. It was later replaced by other versions, the most familiar version being the one formulated in 1845 by Hermann Grassmann (1809–1872), which is today found in basic textbooks. These two laws are still being disputed on their validity because, although they give the same result for the total force between two closed current loops, the distribution of the force along the loops is different, (Assis, 1994).

### 2.3 Ohm's law

After the introduction of the Volta pile, it took three decades before a general understanding was developed on what happened in the circuit connecting the poles. This was mainly because of the instability of the early chemical elements but also due to uncertainty about single-fluid and two-fluid theories. Finally, Georg Simon Ohm published a book in 1827 where the electric current was considered similar to flow of heat. Propagation of heat in a conducting medium had been mathematically analyzed in 1822 by Joseph Fourier (1763–1830) in a revolutionary book (Fourier, 1822) applying partial differential equations and boundary conditions. Ohm related the flow of electricity in a conducting medium to the drop of electroscopic force (potential) whose counterpart is the gradual drop of temperature in a medium for a difference in temperatures at the boundaries (Ohm, 1827).

### 2.4 Weber's force law

Faraday's induction experiment was expressed mathematically in 1845 by Franz Neumann (1798–1865) in terms of a simple quantity known as the vector potential. This expression giving the electromotive force induced in a loop by a changing magnetic field, is known as Faraday's law. In 1846 Wilhelm Weber (1804–1891) was able to unify Coulomb's, Ampère's and Faraday's laws into a single force law. Basically, this was Coulomb's law generalized to two moving point charges and called Weber's force law. To produce Ampère's law, electric current elements were assumed in the form of equal positive and negative charges moving symmetrically in opposite directions so that Coulomb's force was canceled. As a bonus, Weber's formula also gave Faraday's law. Actually, it follows from Ampère's law and energy conservation, as was shown by Helmholtz (Helmholtz, 1827). Because of its dependence on the first and second time derivatives of the distance between the charges, Weber's force law was more general than Newton's law. The critique on failure in energy conservation was proved groundless when Weber derived a generalization of Coulomb's potential function producing the force law. Weber's law gave an instantaneous action-at-a-distance formulation to the basic electromagnetic force and represented the high point of Continental electromagnetics. Retardation due to finite velocity of force was introduced in 1858 by Bernhard Riemann (1826–1866) and in 1867 by Ludwig Lorenz (1829–1891). However, in between Maxwell's theory had already been published.

## 3 British electromagnetics

### 3.1 Faraday's field concept

The British branch of electromagnetics started by Faraday's induction experiment. He explained it in nonmathematical terms by magnetic lines or tubes of force whose changes in a conducting loop would induce an electromotive force. Magnetic lines were visually suggested by figures made by iron filings and he considered them as physical as light rays. Because such lines connect magnetic poles of opposite polarity, they appeared to convey the attractive force like rubber strings. On the other hand, adjacent lines ending to magnetic poles of the same polarity act as tubes under pressure tending to repel each other laterally. Similar force lines were later suggested to the electric force. This kind of physical imagery guided Faraday and his British followers forward. A crucial discovery was made in 1845 when Faraday found that the magnetic field affects the polarization of light in passing through certain media along the magnetic field line. This effect is known today as Faraday rotation. Faraday also predicted in 1832 that magnetic and electric forces are not instantaneous but propagate similarly to sound and light. This he suggested in a sealed letter to be opened only after a century in 1932.

### 3.2 Thomson's analogies

William Thomson (Kelvin) (1824–1907) had read Fourier's book when entering the University of Cambridge in 1840. He wrote a paper showing that Fourier's stationary flow of heat was mathematically analogous to Coulomb's force law even if the former applied contiguous transfer of heat and the latter action over a distance. The lines of heat flow appeared to follow exactly Faraday's electric lines of force. This gave Thomson the idea to represent the electric field in terms of a flux of electricity starting from the charge point. Another set of analogies was found between electrostatic polarization in insulating media and displacements in elastic solids due to stress. In 1856 he made an attempt to explain the Faraday rotation in terms of molecular vortices caused by the magnetic field.

### 3.3 Maxwell's field theory

Thomson's application of analogies found an eager follower in James Clerk Maxwell (1831–1879). In a 1856 article he worked with the analogy of hydrodynamics dealing with an incompressible and massless fluid. Maxwell separated between physical quantities of two kinds, "intensity", like the gradient of the pressure, and "quantity", like the velocity of mass. They obeyed a linear relation involving a parameter of the medium. Thus, for magnetism (electricity) the intensity was denoted by **H** (**E**) and quantity by **B** (**D**) and their relation by the coefficient $\mu$ ($\epsilon$). Applying Stokes' and Gauss' integral theorems Maxwell could express all known basic electromagnetic laws in terms of differential divergence and curl operations (in component form), each relating to some analogy in hydrodynamics. After this, in 1861–1862, he started to construct a single mechanical model to represent the interactions of all electromagnetic phenomena. The starting point was Thomson's idea of representing the magnetic field in terms of rotation. Maxwell interpreted the magnetic intensity **H** as the angular velocity of a vortex of physical fluid whose mass corresponded to the magnetic permeability $\mu$. Separating the vortex tubes by bearing balls, which corresponded to electric charges and conveyed the rotating motion between vortices, he could imitate Ampère's and Faraday's laws in curl equation form. In its first form, the model resembled a clockwork where the motion from vortex to vortex was simultaneous over the whole space. Because it did not yet explain the electrostatic polarization, Maxwell decided to improve the model by adding elasticity to the fluid vortices which corresponded to adding a displacement current term to the existing equations. This made a dramatical change to the stiff clockwork which now started to resemble jelly. In fact, motions were no longer felt simultaneously but they propagated through the structure in a wavelike manner. Computing the velocity of the electromagnetic wave, Maxwell got a result, which was close to the velocity of light known from earlier measurements. This finally unified electromagnetics and light. The displacement-current concept was not based on previous measurements and its experimental verification was delayed by quarter of century because high-frequency equipment was not available at Maxwell's time.

### 3.4 The final touch

Maxwell's final formulation in his book (Maxwell, 1904) involved 20 scalar equations with as many unknowns, which was too complicated for most of his contemporary physicists to grasp. The Maxwell equations we know today in vector form are due to Oliver Heaviside (1850–1925) from 1886 (Heaviside, 1982) as what he called "the duplex method". Heaviside stripped the unnecessary potential quantities from Maxwell's system of equations which left only the field and source quantities in a symmetric form. Heinrich Hertz (1857–1894), who meanwhile had verified Maxwell's displacement current proposition through experiments, also created a corresponding simplied form from Maxwell's formulation. Since then, these equations have found a huge number of applications.

## 4 Conclusions

Evolution of electromagnetics from the Stone age of Coulomb's law to the Modern times of Maxwell's equations took place during the 19th Century. Two main branches could be identified, labeled here as Continental and British electromagnetics. The former had its high point in Weber's force law which unified Coulomb's, Ampère's and Faraday's laws in one neat expression. On the British side, Maxwell constructed a complicated-looking set of equations which also could explain all known electromagnetic phenomena. However, his equations could also predict new phenomena which started a new era in electromagnetics.

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