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Turbo Equalization Of Nonlinear ISI-channels Using High Rate FEC Codes

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Abstract. Turbo equalization is a widely known method to cope with low signal to noise ratio (SNR) channels corrupted by linear intersymbol interference (ISI) (Berrou and Galvieux, 1993; Hagenauer et al., 1997). Recently in this workshop it was reported that also for nonlinear channels a remarkable turbo decoding gain can be achieved (Siegrist et al., 2001). However, the classical turbo equalization relies on code rates at 1/3 up to 1/2 which makes it quite unattractive for high rate data transmission. Considering the potential of iterative equalization and decoding, we obtain a considerable turbo decoding gain also for high rate codes of less than 7% redundancy by using punctured convolutional codes and block codes.

1 Introduction

As linear ISI can be regarded as a special convolutional code, it may be considered as part of a serially concatenated coding scheme. Therefore the concept of iterative equalization and decoding, which is denoted as "turbo equalization", can be applied. Due to the iterative decoding process, the degradation caused by linear ISI can be reduced remarkably or even compensated almost completely (Berrou and Galvieux, 1993; Hagenauer et al., 1997). Furthermore, the performance of the FEC encoded system can be enhanced without an increase in complexity by using an additional ISI channel precoder (Koetter et al., 2004).

Many communication systems are corrupted by nonlinear ISI like e.g. high rate optical data transmission. Non-linearity and ISI are due to propagation effects along the fiber described by the vectorial nonlinear Schroedinger equation and due to the use of photo diodes as signal detectors with square law characteristic.

In general, nonlinear ISI can be described by a finite state nonlinear channel model approximating the received signal after the electrical detection where state tables with conditional probabilities are used, enabling the application of turbo equalization to nonlinear ISI (Siegrist et al., 2001; Sauer-Greff et al., 2001).

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In this paper, we refer to the transmission of digital data over nonlinear channels with constraint length L. Besides the specification of the turbo equalizer, we focus on high data and code rate transmission. In order to increase the code rate with respect to the classical iterative equalization and decoding approach, we first apply punctured convolutional codes with a code rate of 0.933. The possibility of changing the code rate simply by modifying the puncturing scheme could be seen as an advantage of punctured convolutional codes. However, as it is expected from theory, suitable block codes of the same code rate require less SNR for the same bit error ratio (BER). Therefore we also consider a turbo scheme including a block code. Whereas for punctured convolutional codes equalizer and decoder make use of the same maximum a posteriori probability (APP) symbol-by-symbol detecting algorithm, the APP decoder for block codes requires a different approach.

In the following sections we give a short description of the nonlinear channel model based on noiseless, state dependent signal chips as introduced in Siegrist et al. (2001) and present the investigated systems for punctured convolutional and block codes. Simulation results outline the achievable decoding gain by using appropriate turbo equalization.

2 Model of nonlinear channels

For nonlinear channels and additive Gaussian noise the analog front-end that utilizes the nonlinear channel model consists of a bank of filters matched to the q^{L+1} chips (Benedetto et al., 1987; Sauer-Greff et al., 2001), which can be reduced to a minimum effort front-end including a noise limiting lowpass filter with "square root Nyquist" characteristic and a sampler at the symbol rate. However, this leads to a nonrecoverable degradation which in return is small compared to the ideal case (Haunstein et al., 2004).

Assuming stationary or slowly time varying channels, also a probability based nonlinear channel model can be derived, which in contrast is not restricted to signal independent additive Gaussian noise (Siegrist et al., 2001).

The eye diagram of the nonlinear channel shows that the eye is closed at all times, Fig. 1. Without loss of generality the simulations are based on the channel values at sampling phase ϕ=10/32.

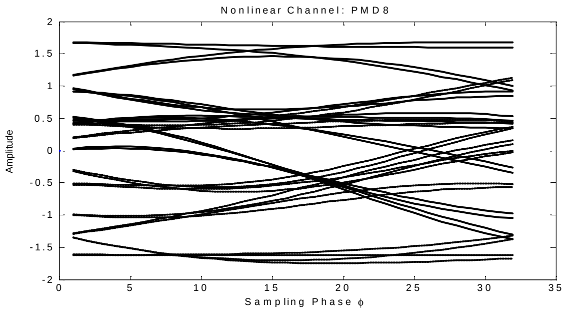

Fig. 1. Eye diagram of nonlinear channel.

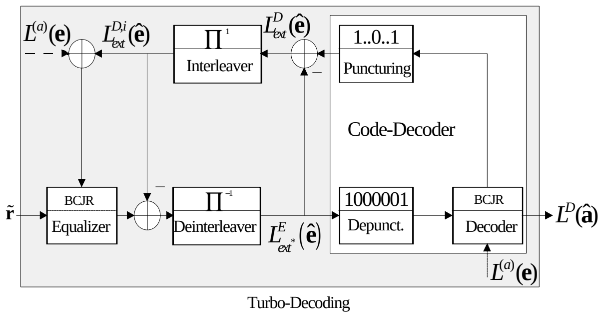

Fig. 2. SCPCC Turbo Decoder. Channel equalizer and decoder are fed with log-likelihood ratios L_{ext} as extrinsic information.

3 Convolutional code turbo equalization

The performance of the serially concatenated punctured convolutional code (SCPCC) turbo scheme in the presence of nonlinear ISI channels is investigated. The channel effects are modeled as mentioned in Sect. 2. An arbitrary bit sequence is encoded by a recursive systematic convolutional (RSC) encoder with code rate $^1/_2$, punctured and transmitted over the nonlinear channel.

The equalizer (Max-Log APP) using a Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm then determines the state transition probabilities referring to the channel output $\tilde{r}_\mu^{(i)}$ by using the conditional probabilities $P\left(r_\mu{=}\tilde{r}^{(i)}\,\middle|\, e_\mu, \mathbf{s}_\mu\right)$ (Siegrist et al., 2001; Sauer-Greff et al., 2001; Bahl et al., 1972). The derived extrinsic soft information is handed over to the code decoder that consists of a puncturing/ depuncturing and a Max-Log APP decoder part in case of convolutional code decoding, Fig. 2.

Equalizer and decoder refer to the received values and additional soft values as in conventional SCC systems. The decoder assumes a log likelihood ratio (LLR) of zero for all punctured bits and refers to the received extrinsic information for all other bits. Since the RSC code with $G_1{=}23_8$ and $G_2{=}35_8$ is systematic, puncturing can be done just by canceling an appropriate number of code bits. Assuming uniform puncturing and a code rate of 0.933 only the first code word consisting of one information and one code bit is transmitted completely. The succeeding 13 code words are transmitted only by half, i.e. only the information bits are sent (Haccoun and Bégin, 1989).

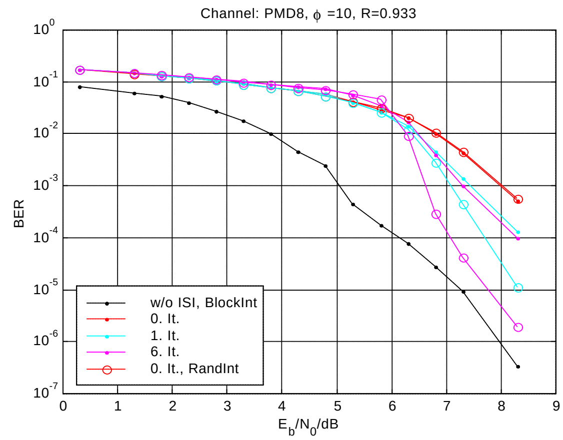

Fig. 3. SCPCC-Turbo-Equalization of a nonlinear optical channel.

A nonlinear optical channel is examined by transmitting 65 025 coded bits with code rate of 0.933 over the channel with memory $M{=}5$. Having the technical implementation in mind, pseudo random as well as block interleavers with the size of 65 025 bits have been used.

The simulation results of the nonlinear channel with a fixed sampling phase and additive Gaussian noise are presented in Fig. 3. It can be seen that the interleaver structure has a remarkable influence on the results. The degradation between an AWGN channel and the nonlinear channel with a pseudo random interleaver is only 0.5 dB after the sixth iteration at a BER of 10^{-5}. Consequently, even in nonlinear systems SCPCC turbo equalization delivers a remarkable decoding gain.

4 Block code soft decoding

Puncturing of convolutional codes to achieve the desired code rates higher than 0.9 results in codes with low performance. A better alternative for these code rates are block codes, e.g. Bose-Chaudhuri-Hocquenghem (BCH) codes, which show a good performance at high code rates using hard decision decoding, like Bounded-Minimum-Distance (BMD) decoding. One drawback of BMD decoding is the restriction to correct less errors than half the designed distance of the code. The designed distance of a BCH-code often remains below the minimum distance, especially at lower code rates.

Another drawback that has to be circumvented is the lack of soft-out information of a BMD decoder, because it is essential in a turbo decoding scheme. Therefore, another decoder has to be found. It is known, that, like convolutional codes, block codes can be represented by a trellis diagram (Wolf, 1978). Using this representation, it is possible to apply the Soft-Out Viterbi Algorithm (SOVA) (Hagenauer and Höher, 1989) or BCJR (Bahl et al., 1972) decoder to use soft-in information and to deliver soft-out information. Unfortunately, this approach demands a prohibitive high complexity decoder, because, e.g., a BCH(255,239) code has a trellis with 2^{16}=65 536 states. Another proposal is to use the dual code for soft-out decoding (Hagenauer et al., 1996), which leads to less, but still quite high effort.

A reduced complexity approach is the modified Chase decoder (Chase, 1972), also known from the decoding of block turbo codes (Pyndiah et al., 1994), as a sub-optimum soft-out decoding method. The Chase algorithm is a list decoding algorithm operating on a code word list. Consider a linear block code with parameters (n, k, δ) and a transmission of binary symbols –1, +1 over a AWGN channel with standard deviation σ. For a transmitted code word $\boldsymbol{e}=(e_0, ..., e_l, ..., e_n)$ the received sequence $\boldsymbol{r}=(r_0, ..., r_l, ..., r_n)$ is given by

$$\boldsymbol{r} = \boldsymbol{e} + \boldsymbol{w}, \tag{1}$$

where $\boldsymbol{w}=(w_0, ..., w_l, ..., w_n)$ are uncorrelated Gaussian noise samples. The optimum decision considering maximum likelihood (ML) decoding is the code word that fulfills the condition

$$\boldsymbol{d} = \boldsymbol{c}^i \text{ with } \left\| \boldsymbol{r} - \boldsymbol{c}^i \right\|^2 \leq \left\| \boldsymbol{r} - \boldsymbol{c}^l \right\|^2, \forall l \in \left[1, 2^k\right], \tag{2}$$

with $\boldsymbol{c}^i=(c^i_0, ..., c^i_l, ..., c^i_n)$ being a code word of the considered block code. At high SNR, the ML code word d is located in a sphere of radius $(\delta-1)$ around the received hard decision word $\boldsymbol{y}=(y_0, ..., y_l, ..., y_n)$, with $y_l=\mathrm{sgn}(r_l)$. I.e., only the code words within this sphere have to be considered in order to find the ML code word. To further reduce the complexity, we take into account only the most probable code words, which are assumed to be given by the positions of the $p=\lfloor \delta/2 \rfloor$ least reliable symbols in $\boldsymbol{r}$. Test patterns $\boldsymbol{t}^q$ are created by toggling the bits of an n-dimensional all-one vector to "–1" at the least reliable positions, starting at the least reliable position with an increasing number of "–1". Feeding all test sequences $\boldsymbol{z}^q$, achieved by component-wise multiplication of $\boldsymbol{y}$ and $\boldsymbol{t}^q$, into an algebraic BMD decoder using e.g. the Berlekamp-Massey-Algorithm (BMA), a set of valid codewords $\boldsymbol{c}^q$ is delivered. The optimum decision $\boldsymbol{d}$ can be found by applying Eq. (2) to all code words $\boldsymbol{c}^q$. The reliability measure is given by log-likelihood ratio (LLR) (Berrou et al., 1993)

$$L(y_i) = \ln\left(\frac{\Pr\left\{e_j = +1 | r_j\right\}}{\Pr\left\{e_j = -1 | r_j\right\}}\right) = \left(\frac{2}{\sigma^2}\right) r_j \,, \tag{3}$$

where the least reliable positions j are the positions with smallest $|r_j|$. For each bit of the hard decision decoder output d, a soft decision in terms of an LLR value has to be found. The LLR value of d_j is given by

$$L(d_i) = \ln\left(\frac{\Pr\left\{e_j = +1 | \boldsymbol{r}\right\}}{\Pr\left\{e_j = -1 | \boldsymbol{r}\right\}}\right). \tag{4}$$

Introducing the sets S_j^{+1} as the set of all code words $\boldsymbol{c}^q$ equal to +1 at position j and the S_j^{-1} as the set of all code words $\boldsymbol{c}^q$ equal to –1 at position j, numerator and denominator of Eq. (4) can be rewritten as sums, resulting in

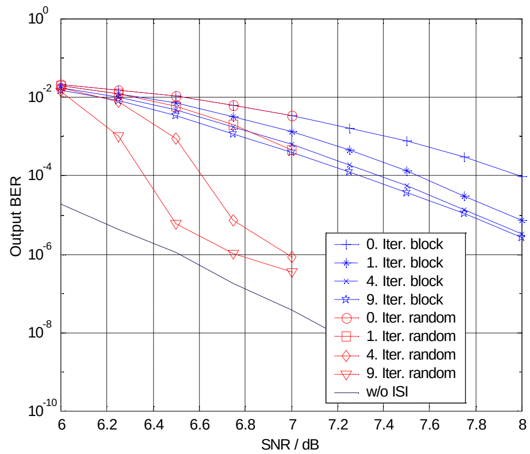

Fig. 4. Block Turbo Equalization of a nonlinear optical channel.

$$L(d_i) = \ln\left(\frac{\sum_{c^q \in S_j^{+1}} p\left\{\boldsymbol{r} | \boldsymbol{e} = \boldsymbol{c}^q\right\}}{\sum_{c^q \in S_j^{-1}} p\left\{\boldsymbol{r} | \boldsymbol{e} = \boldsymbol{c}^q\right\}}\right). \tag{5}$$

A further simplification of Eq. (5) is to consider only the code word in S_j^{+1} and S_j^{-1}, respectively, closest to the received sequence $\boldsymbol{r}$. This eliminates the sums in the numerator and denominator of Eq. (5), avoiding the summation of probabilities given in the logarithmic domain. Obviously (5) delivers LLR values only for positions j for which both S_j^{+1} and S_j^{-1} are non-empty sets. This is not the case for all positions j, if only a finite sphere around the received sequence is considered. One method to overcome this problem is to increase the number of test patterns $\boldsymbol{t}^q$ by increasing the number p of observed least reliable symbols. Hence, more valid code words around the received sequence are obtained, however, requiring a higher decoding effort. Because of this high complexity another solution is chosen. For all j with empty S_j^{+1} or S_j^{-1} a fixed reliability is given with an absolute value β, and we obtain:

$$L(d_i) = \beta \cdot d_j \,. \tag{6}$$

An equation to calculate a nearly optimum β is given by

$$\beta \approx \ln\left(\frac{\Pr\left\{d_j = e_j\right\}}{\Pr\left\{d_j \neq e_j\right\}}\right), \tag{7}$$

where $\Pr\left\{d_j = e_j\right\}$ is the probability of a correct decoder decision for the hard decision bit j. It results in a valid LLR value with SNR dependent magnitude, which might be less accurate than that delivered by Eq. (5).

5 Results of block turbo equalization

Figure 4 shows simulation results using the same channel as investigated with SCPCC Turbo Equalization. A

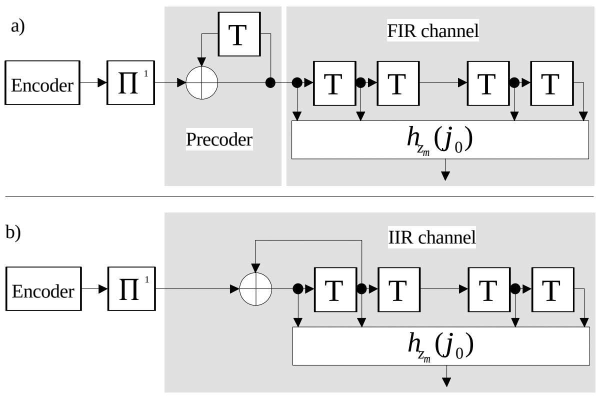

Fig. 5. Usage of a precoder with approaches for **(a)** implementation and **(b)** modeling.

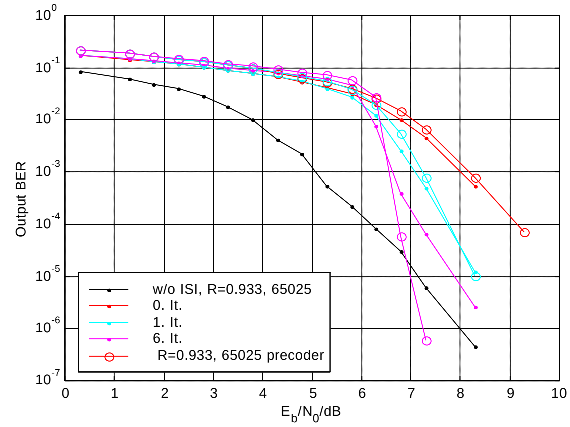

Fig. 7. SCPCC Turbo Equalization with and without precoder.

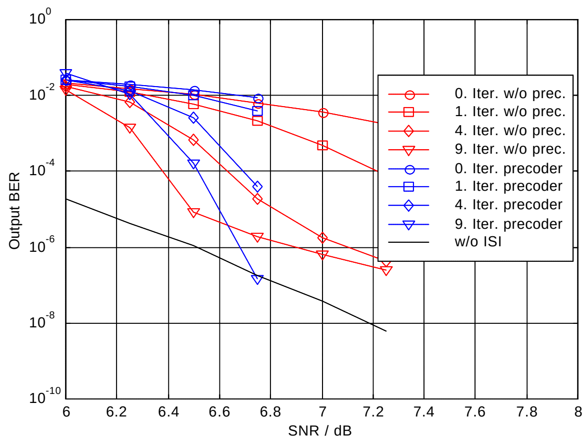

Fig. 6. Block Turbo Equalization with and without precoder.

BCH(255,239)-code with a code rate r=0.937 was applied to a block of length N=65 025. It compares block turbo equalization with block interleaver, random interleaver and ISI-free transmission.

As already mentioned the ISI-free transmission using block codes shows a performance gain versus convolutional codes of 1.5 dB at BER=10^{-6}. Obviously, the usage of simple block interleaving does not lead to high iteration gains. It remains a gap of 1.6 dB between the 9. iteration and the ISI free transmission at BER=10^{-5}. As assumed, a random interleaver gives a performance improvement in terms of higher iteration gains. It is possible to approach the ISI-free bound up to 0.25 dB at BER=10^{-6}.

6 Precoding

The overall properties of the concatenated scheme are strongly dependent on the properties of the component codes. So it might be reasonable that an IIR system as inner code could be advantageous compared to the FIR structure in the case of an ISI channel. As a given channel can not be changed, one way to generate an IIR structure is to apply a recursive precoder in the binary domain. In conjunction with the FIR channel this results in an IIR system (cf. Fig. 5).

Figure 5 shows how this scheme (a) would be implemented and (b) has to be interpreted. The APP detector at the receiver side only has to consider the change of the channel model. It can be easily seen that there is no increase in complexity, since the number of states remains unchanged compared to the system without precoder.

Figure 6 illustrates the performance results of Block Turbo Equalization with precoder compared with the results without precoder. Again, a block length of N=32 640 with the same outer BCH code is used.

Obviously at high SNR the system with precoder delivers an additional gain, which can be explained by an increased minimum distance of the concatenated scheme due to the inner IIR code. It is possible to cut the no ISI bound at 6.75 dB. At low SNR the performance of the precoder system is inferior. Here, the effect has to be considered that single channel errors lead to double bit errors. Note that the region where the precoder shows superior results is at low output BERs.

The application of the precoder in the case of convolutional coding leads to similar results, except that the loss at low SNR is smaller. Figure 7 illustrates the performance results of SCPCC Turbo Equalization of the punctured RSC code with and without precoder.

7 Conclusions

We have shown that “Turbo Equalization”, which is well investigated for linear channels and for low code rates ≤0.5, can also achieve high iteration gains using high rate outer codes on nonlinear channels. This allows for applications in fiber optical transmission, where high code rates are required due to the challenges in high-speed systems.

The performance simulations of punctured convolutional codes and block codes with a code rate of 0.933 for a nonlinear ISI channel have shown that even for strongly punctured convolutional codes a remarkable decoding gain can be achieved. A feasible concept has been presented for iterative equalization of suitable block codes, which have superior performance compared to punctured convolutional codes. Considering the interleaver design, a trade-off between feasibility and performance has to be found, regarding the large gains achieved by the usage of random interleavers.

Finally, without increasing the receiver complexity, a precoding scheme in conjunction with iterative equalization has been presented, which allows for additional performance gain.

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