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Array antennas design in dependence of element-phasing

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Abstract. Array antennas are used in science as well as for commercial and military purposes. The used element antennas act in accordance to their desired uses, for example radars or stationer GPS satellites. Typical components are for example slotted waveguides, patches, yagi-antennas and helix-antennas. All these elements do stand out with their own characteristics based on their special applications. If these elements are formed into an array configuration, the effectiveness can be improved immensely. There is a relation between the array functions and the physical array properties like the element alignment (linear, planar, circular), distances between the elements and so on. Among the physical properties there are other attributes like phase or amplitude coefficients, which are of great significance. The aim of this study was to provide an insight into the problem of array design, as far as the antenna element phase is concerned. Along with this, array radiation characteristics effects are presented. With the help of the extracted cognitions beam forming behaviour can be shown and the array phase behaviour can be analysed. One of the main applications is to simulate the array characteristics, like the radiation characteristic or the gain, for displacements of the array feeding point. A software solution that simulates the phase shift of a given array pattern is sought to adjust the feeding point.

1 Introduction

Nowadays, with SAR (synthetic aperture radar)-applications playing an important role in remote sensing, antenna technology along with others attains significance because of growing bandwidths.

Many antenna design requirements need to be met to achieve a certain functional antenna characteristic. Behind the primary array characteristics like gain, radiation pattern and bandwidth especially the phase behaviour of the array plays an essential role. As the phase behaviour has direct influence on the functionality of the array antenna, it is of significant importance.

The planar array is one possible antenna design for various applications. The choice of the antenna elements depends on several criteria. Besides slotted waveguides, patches, yagi- and helix antennas are used as well. It is very important to pay attention to all array characteristics, especially the phase behaviour, for designing a planar array.

2 Basics

An antenna system is a combination of identical elementary antennas to one antenna group (array). The IEEE Standard 145-1993 quotes:

"Array antenna: An antenna comprised of a number of identical radiating elements in a regular arrangement and excited to obtain a prescribed radiation pattern."

The behaviour of an array depends on the radiation pattern (characteristics) of several element antennas and of the group in a whole. The radiation pattern of the group is formed by ideal, isotropic point sources. Consequently, in practice, the array factor is independent from the used element antennas.

Hence, the multiplication rule can be applied, when all single antennas possess the same characteristics and the behaviour of the array in the far field is considered. (see Balanis *ANTENNA THEORY*)

$$E_t(\text{total field}) = EF(\text{element factor}) \cdot AF(\text{array factor}) \quad (1)$$

3 Array factor of a planar array

The array factor of a planar array is basically calculated by the addition of the characteristics of several isotropic element antennas. In the following, the array factor for an N*M planar array in the x-y-plane shall be derived. Figure 1 schematically shows the structure of an array. The red

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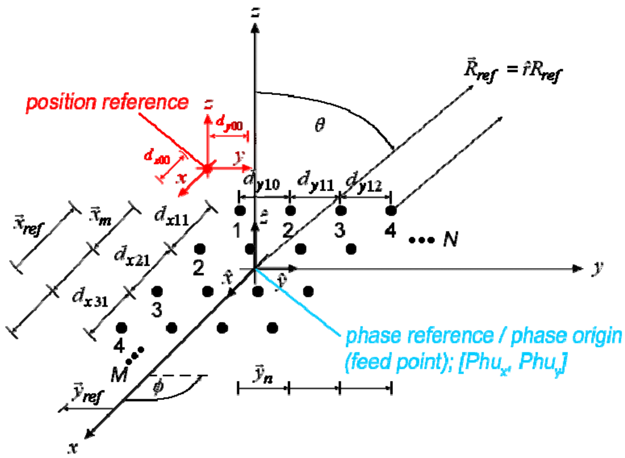

Fig. 1. Diagrammatic structure of a planar array.

marked coordinate system represents the system in which the element antennas are arranged in the x-y-plane. The point of origin is marked by "position reference". Every element antenna is placed in a defined distance $|\boldsymbol{d}_{xmn}|\&|\boldsymbol{d}_{ymn}|$ to its neighbour. The position of the local centre of the array inside the coordinate system (in this configuration: antenna centre = feed point/phase reference/phase origin) can be calculated as follows. The feed point is visualized with the help of the black coordinate system.

$$Phu_x = \frac{1}{N}\sum_{n=1}^{N}\left(0.5\sum_{m=1}^{M-1.}|\boldsymbol{d}_{xmn}|\right) \tag{2}$$

$$Phu_y = \frac{1}{M}\sum_{m=1}^{M}\left(0.5\sum_{n=1}^{N-1.}\left|\boldsymbol{d}_{ymn}\right|\right) \tag{3}$$

A reference distance $\boldsymbol{R}_{\mathrm{ref}}$ can be defined from the antenna centre to a distant object, where, $\boldsymbol{R}_{\mathrm{ref}}$ corresponds to the boresight-direction. Because of the fact, that the antenna elements are arranged in a plane around the antenna centre, they show different distances to the remote object, with respect to the reference distance. These distance differences have to be defined for all elements. For this, the vectors $\boldsymbol{x}_{mn}$, $\boldsymbol{y}_{mn}$, $\boldsymbol{x}_{\mathrm{ref}}$ and $\boldsymbol{y}_{\mathrm{ref}}$ are introduced. $\boldsymbol{x}_{mn}$ describes the respective element distances to the first element in x-direction.

$$\boldsymbol{x}_{mn}=\hat{x}\,(m-1)\,\boldsymbol{d}_{xmn} \text{ for } m=1,2,\ldots,M \tag{4}$$

This applies equally in y-direction

$$\boldsymbol{y}_{mn}=\hat{y}\,(n-1)\,\boldsymbol{d}_{ymn},\ \text{ for } n=1,2,\ldots,N. \tag{5}$$

The vectors $\boldsymbol{x}_{\mathrm{ref}}$ and $\boldsymbol{y}_{\mathrm{ref}}$ describe the distance of the phase reference to the first element in both x- and y-direction.

$$\boldsymbol{x}_{\mathrm{ref}} = \hat{x}\,(Phu_x-1)\,\boldsymbol{d}_{xmn} \tag{6}$$

$$\boldsymbol{y}_{\mathrm{ref}} = \hat{y}\,\left(Phu_y-1\right)\boldsymbol{d}_{ymn} \tag{7}$$

It needs to be kept in mind, that Eqs. (4) to (7) are only valid when all elements show uniform distances between the elements. If this is not the case, the following arithmetic operation needs to be used to adjust $\boldsymbol{d}_{xmn}$ and $\boldsymbol{d}_{ymn}$.

$$\boldsymbol{d}_{xmn} = \frac{\sum_{m\_s=1}^{m}\boldsymbol{d}_{xm\_sn}}{m-1} \tag{8}$$

$$\boldsymbol{d}_{ymn} = \frac{\sum_{n\_s=1}^{n}\boldsymbol{d}_{ymn\_s}}{n-1} \tag{9}$$

Due to the fact, that the phase origin Phu_x and Phu_y is directly dependent on the distance matrix $\boldsymbol{d}_{xmn}$ and $\boldsymbol{d}_{ymn}$, there needs to be another modification. The phase origin Phu_x & Phu_y remains the same, but needs to be transformed into a phase origin matrix $\boldsymbol{Phu}_{xmn}$ and $\boldsymbol{Phu}_{ymn}$ depending on the distance matrices.

$\boldsymbol{Phu}_{xmn}$ and $\boldsymbol{Phu}_{ymn}$ include values that correspond to the correct phase origin value for every $\boldsymbol{d}_{xmn}$ and $\boldsymbol{d}_{ymn}$ accordingly.

The phase origin matrices depending on $\boldsymbol{d}_{xnm}$ and $\boldsymbol{d}_{ymn}$ can be calculated as follows.

$$\boldsymbol{Phu}_{xmn} = \frac{Phu_x}{|\boldsymbol{d}_{xmn}|}+1 \tag{10}$$

$$\boldsymbol{Phu}_{ymn} = \frac{Phu_y}{\left|\boldsymbol{d}_{ymn}\right|}+1 \tag{11}$$

With the help of the preceding equations, a defined distance to a distant object can be referred to every element.

It applies in x-direction

$$\boldsymbol{x}_{\mathrm{ref}}\bullet\hat{r}-\boldsymbol{x}_{mn}\bullet\hat{r}, \tag{12}$$

and in y-direction

$$\boldsymbol{y}_{\mathrm{ref}}\bullet\hat{r}-\boldsymbol{y}_{mn}\bullet\hat{r} \tag{13}$$

with

$$\hat{r}=\hat{x}\sin\theta\cos\phi+\hat{y}\sin\theta\sin\phi+\hat{z}\cos\theta. \tag{14}$$

Apart from the mentioned distance characteristics it is possible to assign a phase shift β_x and β_y to every element. This phase shift depends on the phase reference (feed point) according to the position reference.

The standardised and phase-depending array factor AF can therefore be calculated as follows. $\boldsymbol{A}_{mn}$ stands for the amplitude coefficient of the single elements.

$$AF=\frac{1}{M\cdot N}\sum_{m=1}^{M}\sum_{n=1}^{N}\boldsymbol{A}_{mn}e^{-jk\left[\boldsymbol{x}_{\mathrm{ref}}\bullet\hat{r}-\boldsymbol{x}_{mn}\bullet\hat{r}+\boldsymbol{y}_{\mathrm{ref}}\bullet\hat{r}-\boldsymbol{y}_{mn}\bullet\hat{r}\right]} \tag{15}$$
$$e^{-j(|\boldsymbol{Phu}_{xmn}|-m)\beta_x}e^{-j\left(\left|\boldsymbol{Phu}_{ymn}\right|-n\right)\beta_y}$$

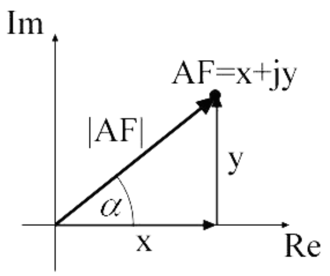

Fig. 2. Real and imaginary parts of the array factor in the Cartesian coordinate system.

After the union of Eqs. (4)–(7) and (14) with Eq. (15), it implies the following simplified equation for the array factor.

$$AF = \frac{1}{M \cdot N} \sum_{m=1}^{M} \sum_{n=1}^{N} \boldsymbol{A}_{mn} \quad (16)$$

$$e^{-j[\psi_x(|\boldsymbol{Phu}_{xmn}|-m)+\psi_y(|\boldsymbol{Phu}_{ymn}|-n)]}$$

with $\psi_{xmn}=k\,|\boldsymbol{d}_{xmn}|\sin\theta\cos\phi+\beta_x$ (17)

$$\psi_{ymn} = k\left|\boldsymbol{d}_{ymn}\right|\sin\theta\sin\phi+\beta_y \quad (18)$$

k corresponds to the phase constant with $k=2\pi/\lambda$. (19)

3.1 Simulating the dependence of the array factor phase to the phase reference (feed point)

The array factor phase:

The array factor of Eq. (16) is characterized by a complex behaviour. It therefore owns a real and an imaginary part.

$$\mathrm{AF} = x + jy \quad (20)$$

This issue is shown in Fig. 2.

Angle α of Fig. 2 corresponds to the argument and therefore to the phase of AF.

$$\alpha = \mathrm{Arg}\,[\mathrm{AF}] = \arctan\left(\frac{y}{x}\right) \quad (21)$$

If Eq. (20) is unified with the polar form, an expression of the array factor directly depending on this argument can be defined.

$$\mathrm{AF} = |\mathrm{AF}|\, e^{j\,\mathrm{Arg}[\mathrm{AF}]} \quad (22)$$

Considering Eq. (21) and (22) it can be said, that the array phase is depending on its array factor and the array factor is, according to Eq. (16), depending on the phase reference $\boldsymbol{Phu}_{xmn}$ and $\boldsymbol{Phu}_{ymn}$; Phu_x and Phu_y.

The array phase can be depicted graphically in different ways. There are the Cartesian, the polar in sectional view

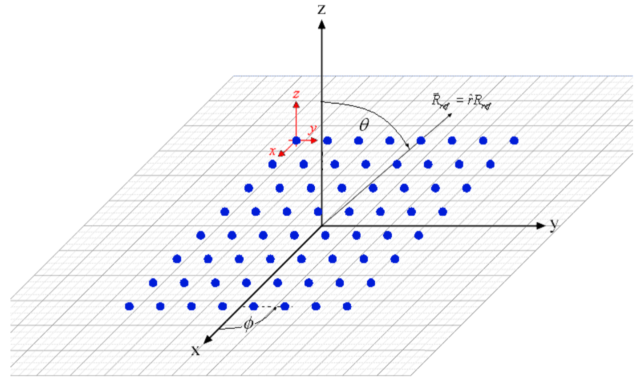

Fig. 3. Visualization of a 8 by 8 point source array in the x-y-plane phase reference [x;y]=Array centre ([1.05 m; 1.05 m] away from first element).

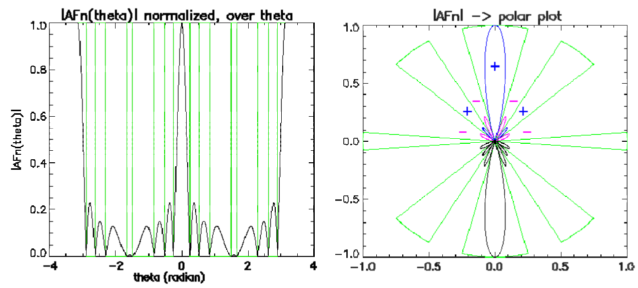

Fig. 4. Cartesian and polar plot of the array factor (black) and the phase (green) with Φ=90°.

and the polar 3-D plot. In the following, the plots shall be illustrated on the basis of a planar 8 by 8 point source array (see Fig. 3). It needs to be kept in mind that the phase is depending on the direction and therefore can be considered from various directions. This is especially shown in the polar sectional view as well as in the 3-D plot.

If the phase reference (feed point of the array) shifts on the x-y-plane due to exterior influences, the phase behaviour of the array is immensely affected. Phase fronts can turn over, shift or distort. The pure antenna pattern (the magnetude of the array factor) remains nevertheless constant. In order to demonstrate the influence that the position of the phase reference has on the array's phase behaviour, the example of an 8 by 8 point source arrays shall be discussed.

Like it was mentioned before, the plots shown in Figs. 3–5 correspond to an array which phase reference is situated in the antenna centre. Now the phase reference (feed point) shall be shifted slightly in x- and y-direction. Figure 6 shows this shift.

Considering Figs. 7 and 8 it becomes clear, that because of the shift of the phase reference, the phase behaviour is changing. The phase fronts shift and reshape.

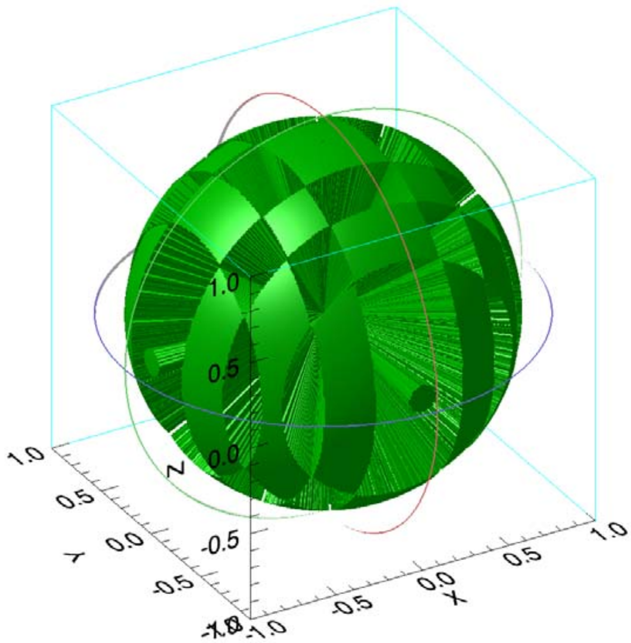

Fig. 5. 3-D plot of the phase of a 8 by 8 point source array.

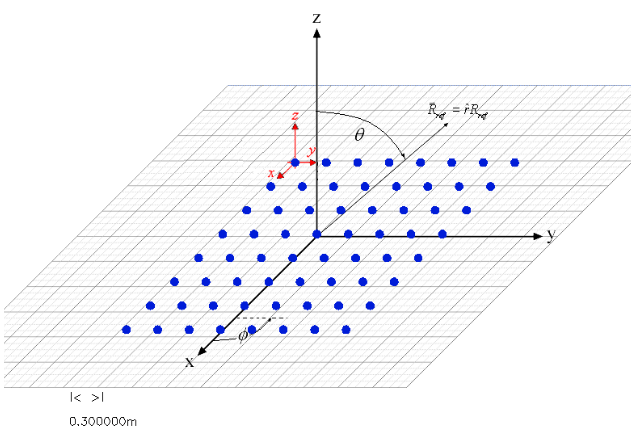

Fig. 6. Visualisation of a 8 by 8 point source array in the x-y-plane phase reference [x;y]=[0.9 m; 0.9 m] away from first element.

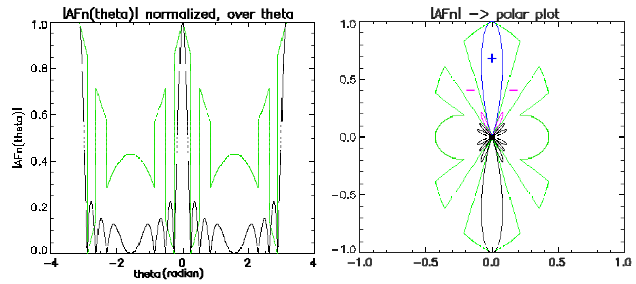

Fig. 7. Cartesian and polar plot of the array factor (black) and the phase (green) with Φ=90° and phase reference [x;y]=[0.9 m; 0.9 m] away from first element.

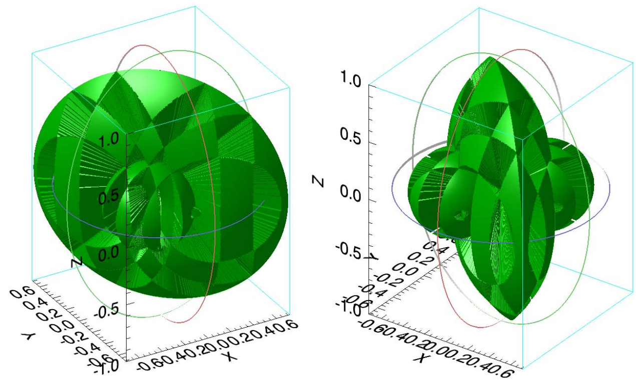

Fig. 8. 3-D plot of the phase of a 8 by 8 point source array with the phase reference [x;y]=[0.9 m; 0.9 m] away from first element.

Furthermore, it can be said, that the phase bias amplifies along with increasing distance between the phase reference and the array centre. Along with this, an increasing phase tapering can be discussed.

It needs to be considered that the phase shows the same behaviour with symmetric positioning of the phase reference around the array centre.

3.2 Phase consideration applications

The consideration of the array phase is useful in many areas, even if it is an artificial array like a synthetic aperture radar (SAR). The SAR makes use of Doppler behaviour, by switching the phase with charging Doppler signal. In order to record the correct Doppler-phase-variations, we need to know the full phase characteristics including shifts, biases, and steep changes. Apart from the control of the phase fronts the simulation of a phase behaviour can be used as well to correct the Doppler signals.

Another use of the phase consideration is the calibration of arrays. The phase is determined by measurement and compared and coordinated with those of the simulation. Along with this, the phase behaviour can be influenced by means of phase shifter and the shifting of the feed point.

A phase consideration can be useful especially for phase arrays. If the phase reference (feed point) is placed in the array centre, the antenna pattern (value of AF) and the phase behaviour are independent from the phase shifts β_x and β_y. With this knowledge a calibration method can be implemented as well. A phase array is combined with arbitrary phase shifts and the antenna pattern as well as the phase behaviour is determined. Accordingly one influences the phase behaviour by shifting the feed point in a way that no phase bias is given and an emission angle of 0° in the direction of elevation and azimuth is present. If this state is achieved, one has localized the antenna centre. Thereby, the simulation shall support the optimization process.

4 Conclusions

The phase behaviour of an array antenna plays an important role next to its antenna pattern and characteristics. The phase position of the element antennas characterizes the phase reference that, intern, again has influence on the characteristics of the array.

The study includes the mathematical modelling of the phase position with planar arrays. By means of simulation, the influences of the phase reference on the phase behaviour could be represented and described. Apart from the analysis of the geometric phase origin, simulation also addresses the examination of the other phase shifts caused by shifts of the feed point or by the use of phase shifters.

The configuration of the feed point of a planar array is of importance because of the direct dependence of the phase behaviour on the array and exerts essential influence on the functionality of different array systems.

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