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Generalizations of Blakesley's Source Shift Theorem

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Abstract. It is shown that the well-known Voltage Source Shift Theorem due to Blakesley and its dual version, the Current Source Shift Theorem as well as the rules for the transformation of networks with loops of capacitors or cut sets of inductors into networks without such loops or cutsets, resp., and the relationships between capacitance coefficients and partial capacitors are special cases of general theorems on the terminal behavior of networks. The proof of these theorems is based on the theory of terminal behavior of networks. For these proofs we do not need the substitution theorem with its strong uniqueness assumptions. This fact is an essential advantage in comparison to the original proof given by Chua and Green.

1 Introduction

The the Voltage Source Shift Theorem (Desoer and Kuh, 1969; Chua, 1987) goes back to the paper of T. H. Blakesley in 1894.

Blakesley considers two cases. In the first one a voltage source is shifted from one branch of an incidence cut into the complementary subset of branches of this cut. In the second one a voltage source is shifted from one branch of an arbitrary cutset into the complementary subset of branches of this cutset.

The Current Source Shift Theorem (Desoer and Kuh, 1969; Chua, 1987) is the dual form of the Voltage Source Shift Theorem. While the latter is definitely connected with the name of Blakesley, we was unable to identify the inventor of the Current Source Shift Theorem.

Chua and Lin consider in Problem 10-9 of Chua and Lin, 1975 a network including a subnetwork which consists of only one loop with four capacitors. It is to show that this subnetwork can be replaced by a tree of three coupled capacitors such that the original and the modified subnetwork have the same terminal behavior. Analogously, they consider there in Problem 10-11 a network including a subnetwork consisting of four inductors which are a cutset of the given network. It is to show that one of these inductors can be replaced by a short circuit while the other three inductors are to be replaced by some coupled inductors such that again the original and the modified subnetwork have the same terminal behavior.

Chua and Green generalize in Chua and Green, 1976 these equivalences to nonlinear RLCM-networks including capacitor voltage-source loops or inductor current-source cutsets.

In almost all of today's circuit simulators the network models to be analyzed are described by means of differential-algebraic equations. The so called index (Hairer and Wanner, 1996; Reich, 1992) describes an essential property of this class of equations. The equivalences considered in Chua and Green, 1976 can be used to reduce the index of the corresponding differential algebraic equation. This is an essential feature for the development of circuit simulation software since index reduction improves the convergence properties of numerical integration procedures for this special type of equations.

For the proof of these equivalences Chua and Green use mathematical induction. To verify the corresponding base step they rely on the Substitution Theorem of network theory (Desoer and Kuh, 1969; Chua, 1987). This fact is the Achilles heel of their proof since the Substitution Theorem can be used only, if both the original network and the network modified by means of this theorem have the same finite number of solutions (cf. Haase, 1985) or, in the standard version (Desoer and Kuh, 1969, Chua, 1987), only unique solutions. Take into account, for nonlinear networks it is impossible to ensure that this assumption is always fulfilled.

The examples of equivalent networks mentioned above are special cases of more general theorems of the terminal behavior of networks. Here we present only the central results and the ideas behind their proofs; a paper with the complete proofs is in preparation (Reibiger, 2009).

For network theoretical notations used in the following we refer to (Reibiger, 2008, 2003a, b). Especially we need here the notions of *constitutive* and *behavioral equations* of a network, of *matrix representations* of the elements of its *universal signal set*, of the *projections* of its *solution set* defined by subsets of its branch set and that of the *canonical representatives* of its *terminal behavior* with respect to a prescribed *terminal class family*.

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The constitutive relation of a network $\mathcal{N}{=}(\mathcal{C},\mathcal{V})$ is denoted as a *conductance-like* constitutive relation if it is not only any binary relation but rather a right-unique relation, i.e. a mapping which assigns to each element u of $\operatorname{dom}\mathcal{V}$ a unique $i{=}\mathcal{V}(u)$. Similarly we refer to $\mathcal{V}$ as an *resistance-like* constitutive relation if $\mathcal{V}$ is a left-unique relation. Then $\mathcal{V}^{-1}$ is a mapping which assigns to each element i of its domain a unique $u{=}\mathcal{V}^{-1}(i)$. It must be emphasized that a network with a conductance- or resistance-like constitutive relation is not necessarily a resistive network and not necessarily a linear one, too.

2 Generalized Current Source Shift Theorems

The Current-Source Shift Theorem (Desoer and Kuh, 1969; Chua, 1987) is one of the classical theorems on the terminal behavior of networks.

In this section we consider a connected network $\mathcal{N}{=}(\mathcal{C},\mathcal{V})$ with associated reference directions, with at least one loop, and without self-loops. We assume that $\mathcal{N}$ consists of two complementary subnetworks $\mathcal{N}^{\mathrm{vs}}$ and $\mathcal{N}^{\mathrm{cl}}$. The subnetwork $\mathcal{N}^{\mathrm{vs}}$ has the branch set $\mathcal{Z}^{\mathrm{vs}}$ and consists of independent voltage sources only, of course, without voltage-source loops. The subnetwork $\mathcal{N}^{\mathrm{cl}}$ has the branch set $\mathcal{Z}^{\mathrm{cl}}$ and a conductance-like constitutive relation. Branch and node set of $\mathcal{N}$ are denoted by $\mathcal{Z}$ and $\mathcal{K}$, resp. Clearly, it holds $\mathcal{Z}{=}\mathcal{Z}^{\mathrm{vs}}\cup\mathcal{Z}^{\mathrm{cl}}$.

Since $\mathcal{N}$ does not include self-loops, each of its loops consists of at least two branches.

Because $\mathcal{N}$ has associated reference directions there exists an oriented graph $\mathcal{G}$ such that $\mathcal{C}{=}(\mathcal{G},\mathcal{G})$.

Since $\mathcal{N}^{\mathrm{vs}}$ is loopless, there exists in $\mathcal{G}$ a spanning tree $\mathcal{G}_{\mathrm{tr}}$ with a branch set $\mathcal{Z}_{\mathrm{tr}}$ consisting of all branches of $\mathcal{Z}^{\mathrm{vs}}$ and a minimal subset $\mathcal{Z}^{\mathrm{cl}}_{\mathrm{tr}}\subset\mathcal{Z}^{\mathrm{cl}}$, i.e., $\mathcal{Z}_{\mathrm{tr}}{=}\mathcal{Z}^{\mathrm{vs}}\cup\mathcal{Z}^{\mathrm{cl}}_{\mathrm{tr}}$. Because $\mathcal{N}$ includes at least one loop, the corresponding set $\mathcal{Z}_{\mathrm{lk}}{:=}\mathcal{Z}^{\mathrm{cl}}_{\mathrm{lk}}{:=}\mathcal{Z}^{\mathrm{cl}}\setminus\mathcal{Z}^{\mathrm{cl}}_{\mathrm{tr}}$ of links is nonvoid and consists merely of branches of $\mathcal{Z}^{\mathrm{cl}}$.

A suitable numbering of the branches of $\mathcal{N}$ presupposed the fundamental cutset matrix of $\mathcal{N}$ defined by $\mathcal{G}_{\mathrm{tr}}$ can be partitioned as follows

$$S=\begin{pmatrix} E^{\mathrm{vs}} & 0 & F^{\mathrm{vs}}\\ 0 & E^{\mathrm{cl}} & F^{\mathrm{cl}}\end{pmatrix}, \tag{1}$$

where E^{vs} and E^{cl} are $|\mathcal{Z}^{\mathrm{vs}}|{\times}|\mathcal{Z}^{\mathrm{vs}}|$ or $|\mathcal{Z}^{\mathrm{cl}}_{\mathrm{tr}}|{\times}|\mathcal{Z}^{\mathrm{cl}}_{\mathrm{tr}}|$ unit matrices, resp.

Using numberings of the branch sets $\mathcal{Z}^{\mathrm{vs}}$, $\mathcal{Z}^{\mathrm{cl}}$, $\mathcal{Z}^{\mathrm{cl}}_{\mathrm{tr}}$, and $\mathcal{Z}^{\mathrm{cl}}_{\mathrm{lk}}$, preserving the arrangement of the corresponding branches introduced by the numbering of $\mathcal{Z}$ to fix the matrix S in Eq. (1) we assign to each signal pair (u,i) of the universal signal set of $\mathcal{N}$ matrix representations of the quantities u, $u_{\mathcal{Z}^{\mathrm{vs}}}$, $u_{\mathcal{Z}^{\mathrm{cl}}}$, $u_{\mathcal{Z}^{\mathrm{cl}}_{\mathrm{tr}}}$, $u_{\mathcal{Z}^{\mathrm{cl}}_{\mathrm{lk}}}$, and i, $i_{\mathcal{Z}^{\mathrm{vs}}}$, $i_{\mathcal{Z}^{\mathrm{cl}}}$, $i_{\mathcal{Z}^{\mathrm{cl}}_{\mathrm{tr}}}$, $i_{\mathcal{Z}^{\mathrm{cl}}_{\mathrm{lk}}}$. These matrix representations are now in the same order denoted by u, u^{vs}, u^{cl}, $u^{\mathrm{cl}}_{\mathrm{tr}}$, $u^{\mathrm{cl}}_{\mathrm{lk}}$ and i, i^{vs}, i^{cl}, $i^{\mathrm{cl}}_{\mathrm{tr}}$, $i^{\mathrm{cl}}_{\mathrm{lk}}$, resp.

Because the subnetwork $\mathcal{N}^{\mathrm{vs}}$ consists of independent voltage sources only and its complementary subnetwork $\mathcal{N}^{\mathrm{cl}}$ has a conductance-like constitutive relation, there exists a column matrix u^{pv} and a column-matrix valued mapping G such that

$$u^{\mathrm{vs}}=u^{\mathrm{pv}}|\operatorname{dom}u^{\mathrm{vs}} \quad\text{and}\quad i^{\mathrm{cl}}=G(u^{\mathrm{cl}}) \tag{2}$$

are constitutive equations of $\mathcal{N}^{vs}$ or $\mathcal{N}^{cl}$, resp. The elements of u^{pv} are the prescribed voltages of $\mathcal{N}^{vs}$ and $i^{\mathrm{cl}}{=}G(u^{\mathrm{cl}})$ is a representation of the constitutive relation of $\mathcal{N}^{\mathrm{cl}}$ in conductance form.

The restriction of u^{pv} to the domain of u^{vs} on the right hand side of Eq. (2) is necessary since in the general case the signal pairs $(u,i)\in\mathcal{V}$ have domains which are proper subintervalls[1] of $\operatorname{dom}u^{\mathrm{pv}}$. But to simplify the notation we skip the appendix "$|\operatorname{dom}u^{\mathrm{vs}}$" in the following.

To determine the terminal behavior of $\mathcal{N}$ with respect to $\mathcal{K}$ we connect $\mathcal{N}$ with a norator network $\tilde{\mathcal{N}}$ with skeleton $\tilde{\mathcal{C}}{=}(\tilde{\mathcal{G}},-\tilde{\mathcal{G}})$ where $\tilde{\mathcal{G}}$ is a tree with node set $\tilde{\mathcal{K}}{:=}\mathcal{K}$ and a branch set $\tilde{\mathcal{Z}}$ with $|\tilde{\mathcal{Z}}|{=}|\mathcal{Z}_{\mathrm{tr}}|$ branches. The branches of this tree are connected parallel to that of $\mathcal{G}_{\mathrm{tr}}$.

Let $\bar{\mathcal{N}}$ denote this interconnection.

For the analysis of $\bar{\mathcal{N}}$ then an appropriate system of behavioral equations is setted up. Elimination of the branch voltages and currents of its subnetwork $\mathcal{N}$ results in a system of constitutive equations of a canonical representative of the terminal behavior of $\mathcal{N}$ defined by the skeleton $(\tilde{\mathcal{G}},\tilde{\mathcal{G}})$. After an exchange of variables we obtain the following theorem.

Theorem 2.1 *Let $\hat{\mathcal{N}}{=}(\hat{\mathcal{C}},\hat{\mathcal{V}})$ denote that canonical representative of the terminal behavior of $\mathcal{N}$ defined by the skeleton $\hat{\mathcal{C}}{:=}(\mathcal{G}_{\mathrm{tr}},\mathcal{G}_{\mathrm{tr}})$.*

Then the constitutive relation of $\hat{\mathcal{N}}$ can be represented with

$$S^{\mathrm{vs}}:=\begin{pmatrix}0 & F^{\mathrm{vs}}\end{pmatrix},\quad S^{\mathrm{cl}}:=\begin{pmatrix}E^{\mathrm{cl}} & F^{\mathrm{cl}}\end{pmatrix}, \tag{3}$$

by means of the system

$$u^{\mathrm{vs}}=u^{\mathrm{pv}},\quad i^{\mathrm{cl}}=S^{\mathrm{cl}}G({}^{\mathrm{t}}S^{\mathrm{cl}}u^{\mathrm{cl}}+{}^{\mathrm{t}}S^{\mathrm{vs}}u^{\mathrm{pv}}) \tag{4}$$

of constitutive equations in hybrid form.

As a canonical representative of the terminal behavior of $\mathcal{N}$ the network $\hat{\mathcal{N}}$ does not include any loops.

3 Generalized Voltage Source Shift Theorems

Blakesley's Voltage Source Shift Theorem (Blakesley, 1994; Desoer and Kuh, 1969; Chua, 1987) is a further example of one of the classical theorems on the terminal behavior of networks.

In this section we consider a network $\mathcal{N}{=}(\mathcal{C},\mathcal{V})$ with associated reference directions, branch set $\mathcal{Z}$, node set $\mathcal{K}$, and

[1]Typical examples for such situations are networks with finite escape times, cf. e.g. Chua and Lin, 1975, p. 442.

a resistance-like constitutive relation. Because $\mathcal{N}$ has associated reference directions, there exists an oriented graph $\mathcal{G}$ such that $\mathcal{C}=(\mathcal{G}, \mathcal{G})$.

In difference to Sect. 2 we consider here the terminal behavior of $\mathcal{N}$ with respect to a prescribed terminal class family $(\mathcal{K}_l)_{l\in L}$ which includes in the general case more than one terminal class. Especially we are here interested in that cutsets of the corresponding interconnections consisting only of branches of $\mathcal{Z}$ and do not partition anyone of the terminal classes $\mathcal{K}_l$ $(l \in L)$. We assume that each such cutset consists of at least two branches.

In difference to Sect. 2 our prior aim is here not the determination of some canonical representatives of the terminal behavior of $\mathcal{N}$ but rather the determination of a network $\bar{\bar{\mathcal{N}}}$ with the same skeleton and the same terminal behavior as the given network $\mathcal{N}$ whereas additionally some branches of $\mathcal{Z}$ are realized in $\bar{\bar{\mathcal{N}}}$ by a short-circuit.

To determine the terminal behavior of $\mathcal{N}$, and later on that of $\bar{\bar{\mathcal{N}}}$, with respect to $(\mathcal{K}_l)_{l\in L}$ we connect their terminal classes with a norator network $\tilde{\mathcal{N}}$ with skeleton $\tilde{\mathcal{C}}=(\tilde{\mathcal{G}}, -\tilde{\mathcal{G}})$ and branch set $\tilde{\mathcal{Z}}$. The graph $\tilde{\mathcal{G}}$ is a forest. The node set of each tree of this forest is equal to exactly one of the terminal classes $\mathcal{K}_l$ $(l \in L)$. The interconnection of $\mathcal{N}$ with $\tilde{\mathcal{N}}$ may be denoted by $\bar{\mathcal{N}}=(\bar{\mathcal{C}}, \bar{\mathcal{V}})$. Let $\bar{\mathcal{G}}_\mathrm{v}$ and $\bar{\mathcal{G}}_\mathrm{c}$ denote its voltage and current graph and $\bar{\mathcal{Z}}$ its branch set.

There exists a minimal subset $\mathcal{Z}_\mathrm{fo} \subset \mathcal{Z}$ such that the subgraphs of the voltage and the current graph of $\bar{\mathcal{N}}$ generated by $\bar{\mathcal{Z}}_\mathrm{fo}:=\tilde{\mathcal{Z}} \cup \mathcal{Z}_\mathrm{fo}$ are spanning forests of these graphs. Let $\mathcal{Z}_\mathrm{lk}:=\mathcal{Z} \setminus \mathcal{Z}_\mathrm{fo}$ denote the corresponding set of links. Using an appropriate numbering of the branches of $\bar{\mathcal{N}}$ there exist a $|\mathcal{Z}_\mathrm{lk}|\times|\tilde{\mathcal{Z}}|$ matrix $\tilde{F}$, a $|\mathcal{Z}_\mathrm{lk}|\times|\mathcal{Z}_\mathrm{fo}|$ matrix F, and a $|\mathcal{Z}_\mathrm{lk}|\times|\mathcal{Z}_\mathrm{lk}|$ unit matrix E such that the matrices

$$\bar{M}_\mathrm{v}=(-\tilde{F} \quad F \quad E), \quad \bar{M}_\mathrm{c}=(\tilde{F} \quad F \quad E) \tag{5}$$

are the fundamental loop matrices of the voltage and current graph of $\bar{\mathcal{N}}$, resp., defined by these spanning forests.

The branches of $\mathcal{Z}_\mathrm{fo}$ are that branches realized in $\bar{\bar{\mathcal{N}}}$ by short-circuits.

By means of the same numbering of the branches of $\bar{\mathcal{Z}}$ used for the determination of the matrices $\bar{M}_\mathrm{v}$ and $\bar{M}_\mathrm{v}$ we assign to each signal pair $(\bar{u}, \bar{\imath})$ of the universal signal set of $\bar{\mathcal{N}}$ a matrix representation. For simplicity this matrix representation is denoted again by $(\bar{u}, \bar{\imath})$.

Using numberings preserving the arrangements of the branches of the subsets $\mathcal{Z}$, $\tilde{\mathcal{Z}}$, $\mathcal{Z}_\mathrm{fo}$, $\mathcal{Z}_\mathrm{lk} \subset \bar{\mathcal{Z}}$ defined by the numbering of the branches of $\bar{\mathcal{Z}}$ we assign to $\bar{u}_{\mathcal{Z}}$, $\bar{u}_{\tilde{\mathcal{Z}}}$, $\bar{u}_{\mathcal{Z}_\mathrm{fo}}$, $\bar{u}_{\mathcal{Z}_\mathrm{lk}}$ and to $\bar{\imath}_{\mathcal{Z}}$, $\bar{\imath}_{\tilde{\mathcal{Z}}}$, $\bar{\imath}_{\mathcal{Z}_\mathrm{fo}}$, $\bar{\imath}_{\mathcal{Z}_\mathrm{lk}}$ the corresponding matrix representations. These matrix representations are denoted in the same order by u, $\tilde{u}$, u_fo, u_lk and i, $\tilde{\imath}$, i_fo, i_lk.

Because $\mathcal{N}$ has a resistance-like constitutive relation there exists a column-matrix valued mapping R such that the constitutive relation of $\mathcal{N}$ can be represented by the following constitutive equation

$$u=R(i) \tag{6}$$

in resistance form.

For the representation of the constitutive relation of $\bar{\bar{\mathcal{N}}}$ by means of a system of constitutive equations we introduce the ansatz

$$u_\mathrm{fo}=0, \quad u_\mathrm{lk}=M\,R({}^\mathrm{t}R\,i_\mathrm{lk})\,. \tag{7}$$

Let $\hat{\mathcal{N}}=(\hat{\mathcal{C}}, \hat{\mathcal{V}})$ and $\check{\mathcal{N}}=(\check{\mathcal{C}}, \check{\mathcal{V}})$ denote the canonical representatives of the terminal behavior of $\mathcal{N}$ or $\bar{\bar{\mathcal{N}}}$, resp.defined by the skeleton $\hat{\mathcal{C}}:=\check{\mathcal{C}}:=(\tilde{\mathcal{G}}, \tilde{\mathcal{G}})$.

The proof that the networks $\mathcal{N}$ and $\bar{\bar{\mathcal{N}}}$ have the same terminal behavior with respect to $(\mathcal{K}_l)_{l\in L}$ is now a little bit more involved then that of the proof of Theorem 2.1 in Sect. 2. This is owed the fact that it is now in the general case impossible to derive for the canonical representatives of these networks constitutive equations in closed form. Nevertheless it can be shown, for details see Reibiger, 2009, that the canonical representatives $\hat{\mathcal{N}}$ and $\check{\mathcal{N}}$ are identical, because their skeletons are identical by definition and their constitutive relations are both equal to the set of all pairs $(\tilde{u}, \tilde{\imath})$ fulfilling for some value of the variable i_lk the equations

$$\tilde{F}\,\tilde{u}=M\;R({}^\mathrm{t}M\,i_\mathrm{lk})\,, \tag{8}$$

$$\tilde{\imath}={}^\mathrm{t}\tilde{F}\,i_\mathrm{lk} \tag{9}$$

and are therefore identical.

In that manner we obtain a proof for the following theorem.

Theorem 3.1 *The networks $\mathcal{N}$ and $\bar{\bar{\mathcal{N}}}$ have the same terminal behavior with respect to the terminal class family $(\mathcal{K}_l)_{l\in L}$.*

In the terminology introduced in Willems, 1991, Poldermann and Willems, 1998 the system of Eqs. (8) and (9) is an example for the representation of a constitutive relation by means of a constitutive equation with a latent variable.

If the branch set $\tilde{\mathcal{Z}}$ of the interconnection of $\mathcal{N}$ with the norator network $\tilde{\mathcal{N}}$ includes a spanning coforest, then the rank of the submatrix $\tilde{F}$ of the matrices $\bar{M}_\mathrm{v}$ and $\bar{M}_\mathrm{v}$ is equal to $|\mathcal{Z}_\mathrm{lk}|$. Under this assumption it is without additional assumptions on the properties of the column-matrix valued mapping R possible to eliminate the latent variable i_lk included in Eq. (8). This elimination results in a constitutive equation for $\hat{\mathcal{N}}$ hybrid form.

In Reibiger, 2009 a generalization of Theorem 3.1 is proved wherein for $\mathcal{N}$ networks are admitted which include additionally subnetworks consisting of independent current sources.

4 Concluding remarks

We have presented generalizations of the well-known Source Shift Theorems.

Theorem 2.1 includes as special cases both the Current Source Shift Theorem and some of the examples discussed in Chua and Lin, 1975. Since the constitutive relations of nondegenerated linear and nonlinear capacitor networks can be represented by conductance-like constitutive equations, the theorem of Chua and Green (Chua and Green, 1976) for the elimination of loops consisting of coupled capacitors and independent voltage sources is another special case of Theorem 2.1.

Theorem 3.1 includes as special cases both Blakesley's Voltage Source Shift Theorem and some of the examples discussed in Chua and Lin, 1975. Since the constitutive relations of nondegenerated linear and nonlinear inductor networks can be represented by resistance-like constitutive equations, the theorem of Chua and Green (Chua and Green, 1976) for the elimination of cutsets consisting of coupled inductors and independent current sources is also a special case of Theorem 3.1 and its generalization proved in Reibiger, 2009.

The proofs of these theorems are obtained by a unified method based on a theory of terminal behavior of networks developed in Reibiger, 1985, 1986, 2003. The interconnection of the networks under consideration with trees or forests of norators are substantial parts of this approach. Other applications of this method are to find in Reibiger, 1986, 1997, 2008; Reibiger et al., 2003. By the way, these results show that norators (and nullators, too) are by no means "pathological" objects in network theory since their introduction simplifies and unifies the representation of network theory and delivers even starting points for developing important analysis methods.

The most essential applications of the Theorems 2.1 and 3.1 are their use for the reduction of the index of differential-algebraic equations (Reich, 1992) for the analysis of RLCM networks by the elimination of voltage source-capacitor loops and current source-inductor cutsets.

For index reduction by means of an elimination of the voltage source-capacitor loops of any RLCM network by means of Theorem 2.1 the network $\mathcal{N}$ introduced in Sect. 2 has to be that subnetwork of the given network which includes all voltage sources and capacitors which form such loops. Clearly, it can be thereby not permitted that some of the terminals of this subnetwork are in its complementary "external" network directly connected by capacitors or voltage sources.

Similarly, for index reduction by means of an elimination of the current source-inductor cutsets of any RLCM network by means of Theorem 3.1 the network $\mathcal{N}$ introduced in Sect. 3 has to be that subnetwork of the given network which includes all current sources and inductors which form such cutsets.

A simple algorithm for the detection of the corresponding subnetworks of a given network is described in Pottle, 1966. This algorithm is based on the transformation of the incidence matrix of the voltage (or current) graph of the network under consideration into a row-echelon form.

If $\mathcal{N}$ is a capacitor network with n nodes whose voltage graph is a complete graph (cf. Thulasiraman and Swamy, 1992) with $(n+1)n/2$ branches, then it is by Theorem 2.1 possible to replace this network by a canonical representative $\hat{\mathcal{N}}$ consisting of n coupled capacitors whose voltage graph is a star-like tree. This transformation is the inverse of a classical transformation (Sommerfeld, 1988, Meetz and Engl, 1980, Leuchtmann, 2005) assigning to the matrix of capacitance coefficients introduced by J. C. Maxwell, (1980, art. 87), a network consisting of so called partial capacitors since the matrix of capacitance coefficients can be directly interpreted as the coefficient matrix of a system of constitutive equations of a network with a star-like voltage graph consisting of coupled capacitors Meetz and Engl, (1980, 482 pp.). Both kinds of capacitive networks can be used as models for physical multi-electrode capacitors. Yet, from the point of view of the theory of differential-algebraic equations as well as the theory of state-space equations it follows that for modeling of a physical multi-electrode capacitor there are generally network models based on a tree of coupled capacitors to prefer over that based on a complete graph of uncoupled capacitors.

The proof of Theorems 3.1 is not a dualization of the proof of Theorem 2.1, resp. It seems to be of interest whether in the theory of graphoidale networks (Reibiger and Loose, 2007) such a dualization is possible. However for this purpose it would be first necessary to develop for this class of generalized networks a theory of multiport behavior as a counterpart to the theory of terminal behavior considered in Reibiger, 2003a.

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