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Using quasi-guided modes for modeling the transfer behavior of bent dielectric slab waveguides

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Abstract. The connection of two straight dielectric multimode slab waveguides by a circular bent waveguide is analyzed by means of quasi-guided modes. These modes correspond to the well known leaky modes, but own real eigenvalues, thus the mathematical description is simpler. Furthermore they are derived as approximate solutions of the exact theory. This work will first give a brief introduction to the basic theory, followed by a discussion of the properties of quasi-guided modes. After a validation by comparison with a numerical simulation using the Finite Integration Technique, results for the bending loss of multimode waveguides are presented.

1 Introduction

The modeling of bending loss of optical waveguides has been of special interest since the demonstration of the first functional laser in 1960. Many classical publications exist, e.g. Marcatili (1969); Lewin et al. (1977); Marcuse (1982); Snyder and Love (2000), but the presented approaches are mostly approximations for small radiation loss. Often the leaky mode ansatz is presented as exact solution of Maxwell's equations for the cylindrical slab waveguide. This ansatz uses Hankel functions in the outer region in order to satisfy the radiation condition. But it is not the single mode that has to satisfy this condition, it is the sum of all modes. In addition the discrete and limited set of leaky modes can not build a complete set of solutions. In 1990 Morita and Yamata presented a new, initially nonrestrictive theory, but also applied the radiation condition to the individual mode (Morita and Yamada, 1990). Hence it was Kerndlmaier in 1992 who first presented the theory, that is not limited to any special case. Thus he proposed it to be the exact theory (Kerndlmaier, 1992). To the best of the authors knowledge Kerndlmaier did not publish his results apart from his PhD-thesis, thus Sect. 2 deals with the basic ideas of his theory.

The mode spectrum of the bent waveguide is continuous, like the set of radiation modes of the straight slab waveguide. As the primal purpose of our approach is to estimate the radiation loss of bent waveguides connected with straight waveguides, that only carry guided modes at incoming side, it would be advantageous if equivalent modes exist for the bent waveguide. Actually it is possible to isolate a set of quasi guided modes that correspond to the set of leaky modes, but own real eigenvalues. Section 3 characterizes the properties of the quasi-guided modes and describes the connection to straight waveguide elements. For the purpose of validation, results of a numerical simulation using the Finite Integration Technique are presented in Sect. 4. Finally Sect. 5 presents results for the bending loss of multimode slab waveguides involving a 90° bend.

Throughout the following presentation only the case of transverse electric fields is regarded explictly. But all statements also hold for transverse magnetics fields.

2 Basic theory

The following sections briefly describe the theory of quasi-guided modes of bent dielectric slab waveguides. A detailed presentation including all derivations can be found in Kerndlmaier (1992); Stallein (2010).

2.1 General solution of Maxwell's equations

Consider the waveguide structure shown in Fig. 1. The different sections, divided by the inner radius R^- and outer radius R^+, are free of sources and filled with homogeneous material. The refractive indices in the inner cladding, the

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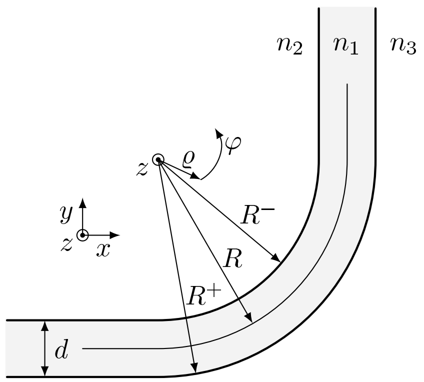


Fig. 1. Connection of two straight waveguides by a bent element.

outer cladding and the core region are n_2, n_3, and n_1, respectively, while the permeability remains constant $\mu = \mu_0$. Using cylindrical coordinates (ϱ, φ, z) the straightforward solution of Maxwell's equations for the bent section in the frequency domain is

$$\underline{\boldsymbol{E}}_\nu(\boldsymbol{r}) = a_\nu \boldsymbol{e}_z \underline{\mathcal{E}}_\nu(\varrho)\, \exp(-j\nu\varphi) \tag{1}$$

with

$$\underline{\mathcal{E}}_\nu(\varrho) = \begin{cases} J_\nu(k_2\varrho) & , 0 \le \varrho \le R^- \\ A_{1\nu} J_\nu(k_1\varrho) + A_{2\nu} N_\nu(k_1\varrho) & , R^- \le \varrho \le R^+ \\ A_{3\nu} J_\nu(k_3\varrho) + A_{4\nu} N_\nu(k_3\varrho) & , R^+ \le \varrho \end{cases} \tag{2}$$

and $k_i = n_i \frac{2\pi}{\lambda}$, $i = 1,2,3$. Here a wave propagation in positive φ-direction has been assumed and ν is called "angular wavenumber" or simply eigenvalue of the mode. J_ν and N_ν are the Bessel and Neumann functions, and λ is the free space wavelength. The magnetic field is given by Faraday's law $\underline{\boldsymbol{H}}_\nu = -1/(j\omega\mu)\nabla \times \underline{\boldsymbol{E}}_\nu$.

Enforcing the boundary conditions in $\varrho = R^-$ and $\varrho = R^+$ results in the following expressions for $A_{1\nu}$–$A_{4\nu}$.

$$A_{1\nu} = \frac{\pi R^-}{2}\Big(k_1 J_\nu(k_2R^-)N'_\nu(k_1R^-) - k_2 J'_\nu(k_2R^-)N_\nu(k_1R^-)\Big) \tag{3}$$

$$A_{2\nu} = \frac{\pi R^-}{2}\Big(k_2 J'_\nu(k_2R^-)J_\nu(k_1R^-) - k_1 J_\nu(k_2R^-)J'_\nu(k_1R^-)\Big) \tag{4}$$

$$A_{3\nu} = \frac{\pi R^+}{2}\Big(-k_1 N_\nu(k_3R^+) \big(A_{1\nu} J'_\nu(k_1R^+) + A_{2\nu} N'_\nu(k_1R^+)\big) + k_3 N'_\nu(k_3R^+) \big(A_{1\nu} J_\nu(k_1R^+) + A_{2\nu} N_\nu(k_1R^+)\big)\Big) \tag{5}$$

$$A_{4\nu} = \frac{\pi R^+}{2}\Big(k_1 J_\nu(k_3R^+) \big(A_{1\nu} J'_\nu(k_1R^+) + A_{2\nu} N'_\nu(k_1R^+)\big) - k_3 J'_\nu(k_3R^+) \big(A_{1\nu} J_\nu(k_1R^+) + A_{2\nu} N_\nu(k_1R^+)\big)\Big) \tag{6}$$

The property a_ν remains as initially undetermined weight function of the continuous mode spectrum of the bent waveguide.

Because of the analogy to the spectrum of radiation modes of the straight waveguide one can assume that purely real eigenvalues ν are adequate to describe any essential physical field problem (Kerndlmaier, 1992), although in general any complex eigenvalue ν is permitted. Therefore in the following only purely real eigenvalues ν are considered and for further characterization of the mode spectrum a scaling of the weight function

$$a_\nu = \sqrt{\frac{2\omega\mu}{A_{3\nu}^2 + A_{4\nu}^2}} \tag{7}$$

is useful.

2.2 Non-orthogonality of the mode spectrum

Testing the orthogonality requires the solution of an integral like

$$\overline{P}'_{\nu\xi} = \frac{\overline{P}_{\nu\xi}}{\Delta z} = \frac{1}{4}\int_0^\infty \Big(\underline{\boldsymbol{E}}_\nu \times \underline{\boldsymbol{H}}_\xi^* + \underline{\boldsymbol{E}}_\xi^* \times \underline{\boldsymbol{H}}_\nu\Big)_\varphi \mathrm{d}\rho \tag{8}$$

where Δz is a distance in z-direction. According to Morita and Yamada (1990) the solution of Eq. (8) is

$$\overline{P}'_{\nu\xi} = \Im\left\{\frac{(A_{3\nu} + jA_{4\nu})}{\sqrt{A_{3\nu}^2 + A_{4\nu}^2}} \frac{(A_{3\xi} - jA_{4\xi})}{\sqrt{A_{3\xi}^2 + A_{4\xi}^2}} \frac{e^{j\frac{\pi}{2}(\nu-\xi)}}{\pi(\nu-\xi)}\right\}. \tag{9}$$

Using the shortcut $\beta_\nu = \frac{A_{3\nu}}{A_{4\nu}}$ it is straightforward to show that no singularity exists for $\nu = \xi$:

$$\overline{P}'_{\nu\nu} = \frac{1}{2} - \frac{1}{\pi}\frac{\mathrm{d}\beta_\nu}{\mathrm{d}\nu}. \tag{10}$$

The expression (9) does not vanish for any $\nu \ne \xi$,

$$\overline{P}'_{\nu\xi} \ne 0, \tag{11}$$

thus the modes of the bent waveguide are nonorthogonal.

2.3 Quasi-guided modes (QGM)

In the continuous mode spectrum of the bent waveguide solutions exist, whose field profiles are focused on the core region. Due to the singularity of the Neumann function N_ν this is the case if

$$A_{3\nu} = 0. \tag{12}$$

If only quasi-guided modes are used in Eq. (9), the expression simplifies with $\beta_\nu = \beta_\xi = 0$ to

$$\overline{P}'_{\nu\xi} = \frac{\sin(\frac{\pi}{2}(\nu-\xi))}{\pi(\nu-\xi)} \qquad \text{if} \quad \nu \neq \xi. \tag{13}$$

As will be shown, in case of quasi-guided modes $\frac{\mathrm{d}\beta_\nu}{\mathrm{d}\nu}$ is always a large negative quantity, thus

$$\overline{P}'_{\nu\nu} \gg \overline{P}'_{\nu\xi} \tag{14}$$

is valid and the quasi-guided modes are considered as quasi-orthogonal, though the complete mode spectrum is non-orthogonal.

2.4 Physical fields

The modes of the bent waveguide are exited by the fields of a straight waveguide, see Fig. 1. If the fields at the end of the straight waveguide in $\varphi = 0$ are given by $\underline{\boldsymbol{\mathcal{E}}}$ and $\underline{\boldsymbol{\mathcal{H}}}$, then the overlap with the bent waveguide modes results in an integral equation of the form

$$I_\xi = \int_\nu a_\nu \overline{P}'_{\nu\xi} d\nu \tag{15}$$

with

$$I_\xi = \frac{1}{4}\int_0^\infty \Big(\underline{\boldsymbol{\mathcal{E}}} \times \underline{\boldsymbol{H}}^*_\xi + \underline{\boldsymbol{E}}^*_\xi \times \underline{\boldsymbol{\mathcal{H}}}\Big)_\varphi \mathrm{d}\rho. \tag{16}$$

Equation (15) is a Fredholm integral equation of the first kind, that is even numerically difficult to solve.

For the connection of straight and bent waveguides it is expected that primarily the codomain for ν next to the eigenvalues ν_0 of the quasi-guided modes is excited. Thus a linearization of $\beta_\nu = \frac{A_{3\nu}}{A_{4\nu}}$ around the eigenvalue ν_0 of a quasi-guided mode is possible

$$\bar{\beta_\nu} = -k\bar{\nu} \text{ with } \bar{\nu} = \nu - \nu_0 \text{ and } -k = \frac{\mathrm{d}\beta_\nu}{\mathrm{d}\nu}\Big|_{\nu_0}. \tag{17}$$

The quantity k is always positive. Now Eq. (9) becomes

$$\overline{P}'_{\nu\xi} = \frac{1}{\pi}\Bigg(\frac{k}{\sqrt{(1+k^2\bar{\nu}^2)(1+k^2\bar{\xi}^2)}}\cos(\tfrac{\pi}{2}(\bar{\nu}-\bar{\xi})) + \frac{1+k^2\bar{\nu}\bar{\xi}}{\sqrt{(1+k^2\bar{\nu}^2)(1+k^2\bar{\xi}^2)}}\frac{\sin(\frac{\pi}{2}(\bar{\nu}-\bar{\xi}))}{\bar{\nu}-\bar{\xi}}\Bigg). \tag{18}$$

If the kernel (18) is used in the integral equation (15), then the square root of a Lorentz distribution ($\sqrt{\text{Lorentz}}$-Distribution) is an eigensolution of the integral equation

$$\int_\nu \frac{1}{\sqrt{1+k^2\bar{\nu}^2}}\overline{P}'_{\nu\xi}\,\mathrm{d}\nu = \frac{1}{\sqrt{1+k^2\bar{\xi}^2}}. \tag{19}$$

This can be checked by means of the residue theorem.

Now let us assume the fields in the bent waveguide are defined by a weight function of the form of a $\sqrt{\text{Lorentz}}$-distribution, thus

$$\underline{\boldsymbol{E}}(\varrho,\varphi) = \int_\nu a_\nu \boldsymbol{e}_z \underline{\mathcal{E}}_\nu(\varrho)\exp(-j\nu\varphi)\mathrm{d}\nu$$

$$\text{with} \quad a_\nu = \frac{1}{\sqrt{1+k^2\bar{\nu}^2}}. \tag{20}$$

When the wave travels through the bent waveguide the shape of the field profile will undergo deformation. As a measure for the deformation after the wave has passed an arbitrary angle φ, the following integral will calculate the overlap with the fields in the plane $\varphi = 0$

$$B(\varphi) = \frac{1}{4}\int_0^\infty \Big(\underline{\boldsymbol{E}}(\varrho,0)\times\underline{\boldsymbol{H}}^*(\varrho,\varphi) + \underline{\boldsymbol{E}}^*(\varrho,\varphi)\times\underline{\boldsymbol{H}}(\varrho,0)\Big)_\varphi \mathrm{d}\rho. \tag{21}$$

Using Eqs. (20), (8) and (19) it follows

$$B(\varphi) = \int_\xi \frac{1}{1+k^2\bar{\xi}^2}\exp(j\xi\varphi)\mathrm{d}\xi. \tag{22}$$

If the integration interval can be extended from the real axis to the closed contour of the upper half plane, it is possible to find a solution by means of the residue theorem. The normalized result is

$$\frac{B(\varphi)}{B(0)} = \frac{1}{\pi k}\oint\limits_{\substack{\text{upper}\\\text{half plane}}} \frac{\exp(j\xi\varphi)}{(\bar{\xi}-j/k)(\bar{\xi}+j/k)}\mathrm{d}\xi = \exp(-\varphi/k). \tag{23}$$

Therefore, as long as $k \gg \varphi$, the field profile defined by a $\sqrt{\text{Lorentz}}$-distribution propagates almost undistorted through the waveguide.

2.5 Relation to the Leaky Mode Ansatz (LMA)

With

$$A_{3\nu} = jA_{4\nu} \tag{24}$$

the field description in $\varrho > R^+$ can be written in terms of Hankel functions

$$A_{3\nu}J_\nu(k_3\varrho) + A_{4\nu}N_\nu(k_3\varrho) = jA_{4\nu}H^{(2)}_\nu. \tag{25}$$

This is the so called Leaky Mode Ansatz (LMA) and Eq. (24) is the associated eigenvalue equation. The Solutions ν_l of Eq. (24) are always complex valued with $\Re\{\nu_l\} \neq 0$ and $\Im\{\nu_l\} \neq 0$. Hence the real part of the Poynting vector of a leaky mode has a component in ϱ-direction.

If $-1/k$ is taken as imaginary part of the eigenvalue ν_0 of the quasi-guided mode, then this constructed eigenvalue can be checked against the eigenvalues of the leaky mode ansatz.

Table 1. Eigenvalues and attenuation constants of quasi-guided modes (ν_0 and $-1/k$) and leaky modes ($\Re\{\nu_l\}$ and $\Im\{\nu_l\}$). Parameters: $n_1 = 1.57$, $n_2 = n_3 = 1.55$, $d = 10\mu$m, $R = 3$mm and $\lambda = 850$nm.

	$\Re\{\nu\}$		$\Im\{\nu\}$	
Mode	QGM	LMA	QGM	LMA
1	34820.05	34820.05	−6.199e-34	–
2	34772.80	34772.80	−6.427e-28	–
3	34720.68	34720.68	−1.284e-21	–
4	34650.55	34650.55	−4.565e-14	–
5	34561.82	34561.82	−2.961e-6	−2.957e-6
6	34460.38	34460.58	−1.308	−1.263

Table 1 shows the results for a waveguide with six guided modes in case of the straight waveguide, and also six quasi-guided modes in case of the bent-waveguide. Except for the last mode the real parts agree within seven (or more) digits. The real parts of the sixth mode differ slightly but now the attenuation constants, the imaginary parts of the eigenvalues, already take large (negative) values, thus the mode suffers strong attenuation. The absolute value $|1/k|$ is always a little larger then $|\Im\{\nu_l\}|$. But anyway, for the first four modes the absolute values are very small and can be neglected in case of waveguide bends of 90° or less. Effectively the quantity $|\Im\{\nu_l\}|$ is too small to be determined by conventional root solvers and double precision accuracy, e.g. secant methods.

The results of Table 1 are typical and similar conclusions hold for waveguides with a larger number of guided modes. There is always a specific number of modes in dependency on the radius R, whose attenuation constant is negligible. Only a few modes remain with $-1/k$ in a significant range and all other modes can be disregarded at all, because the attenuation is to high. The differences in the eigenvalues of the quasi-guided modes and the leaky modes are always very small. Since the theory of quasi-guided modes owns a real valued eigenvalue equation (12), it is preferable, because the leaky modes afford the solution of a complex valued Eq. (24).

Finally Fig. 2 shows the electric field profile of the fifth and sixth mode of the quasi-guided mode approach, the real and imaginary part of the leaky mode ansatz, and the related profile of the mode of the straight waveguide. All fields are normalized with respect to the power flow in φ-direction. For the fifth mode both bent waveguide solutions approximately coincide and the overlap with the mode of the straight waveguide is high. In case of the sixth mode the leaky mode ansatz exhibits an apparent imaginary part. Obviously this mode cannot be regarded as quasi-guided because the oscillations in the outer cladding region $R^+ < \varrho$ are oversized. Anyway, according to Table 1 the attenuation constant of this mode is very high.

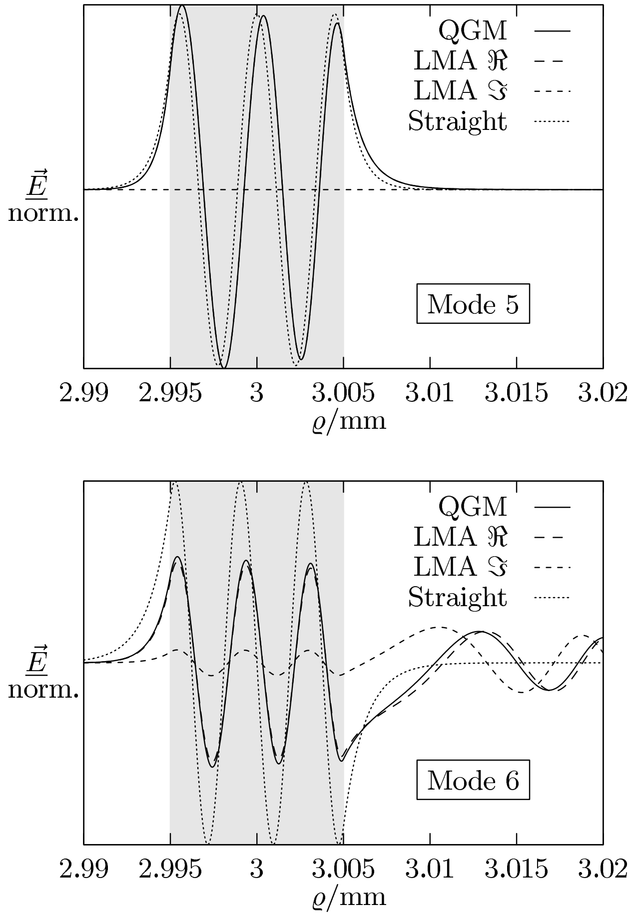


Fig. 2. Comparison of the electric field profile of the fifth and sixth mode of the quasi-guided mode approach (QGM), real and imaginary part of the leaky mode ansatz (LMA $\Re$ and LMA $\Im$), and the related profile of the mode of the straight waveguide (Straight). The waveguide core region is highlighted. Parameters like in Table 1.

3 Calculations on the basis of quasi-guided modes

3.1 Further properties of quasi-guided modes

In Sect. 2.4 the $\sqrt{\text{Lorentz}}$-distribution has been introduced as a physical solution for the weight function a_ν. Because k is a large quantity and $2/k$ is the half width of the Lorentz-distribution, the half width of the first five modes in Table 1 is negligible compared to the distance between the eigenvalues. Even for the sixth mode the distribution concentrates around the quasi eigenvalue. This behavior has been observed for various different waveguide parameters, i.e. different core thicknesses d and radii of curvature R. Therefore it is legitimate to state that the width of the $\sqrt{\text{Lorentz}}$-distribution is infinitesimal small, thus replace it by a Dirac-distribution δ. Then for a multimode waveguide a_ν becomes

$$a_\nu \approx \sum_n C_n \exp(-\alpha_n \varphi)\,\delta(\nu - \nu_n). \tag{26}$$

Here C_n is the amplitude, ν_n the eigenvalue, and $\alpha_n = 1/k_n$ the attenuation constant of a quasi guided mode. Furthermore the electric field is expressed by

$$\underline{\boldsymbol{E}}(\varrho,\varphi) = \int_\nu a_\nu \underline{\boldsymbol{\mathcal{E}}}_\nu(\varrho)\exp(-j\nu\varphi)d\nu$$
$$= \sum_n C_n \underline{\boldsymbol{\mathcal{E}}}_n(\varrho)\exp\big(-j(\nu_n - j\alpha_n)\varphi\big). \qquad (27)$$

As will be shown later, this is quite a good approximation whenever the waveguide bend is free of loss due to radiation.

The connection of the eigenvalues of the straight and the bent waveguide becomes obvious if the propagation terms are compared:

$$\exp(-jk_z z) \approx \exp(-jk_z R\varphi) \quad \Rightarrow \quad \nu \approx k_z R. \qquad (28)$$

Here k_z is the eigenvalue of the straight waveguide. For small radii this relation is just a guesstimate, but it is useful for the definition of a search interval. Based on the well know solution interval of the straight waveguide it follows

$$k_0 n_2 < k_z < k_0 n_1 \quad \Rightarrow \quad k_0 n_2 R^- < \nu < k_0 n_1 R^+. \qquad (29)$$

However, the eigenvalue ν of the fundamental mode might slightly exceed the upper boundary in Eq. (29).

3.2 Connection to straight multimode waveguides

If reflected waves can be neglected at the transition from the straight to the bent waveguide only the transverse components of the electric or the magnetic fields must be matched. Thus if the fields in the bent waveguide are given by Eq. (27), then the amplitudes of the quasi-guided modes are

$$C_n^{(b)} \approx \frac{1}{2\overline{P}'^{(b)}_{nn}} \int_0^\infty \left(\sum_\mu C_\mu^{(l)} \underline{\boldsymbol{\mathcal{E}}}^{(l)}_{t\mu}\right) \times \underline{\boldsymbol{\mathcal{H}}}^{(b)}_{tn}\, \mathrm{d}\rho. \qquad (30)$$

The index (b) indicates the bent waveguide and (l) the straight waveguide. Furthermore the index μ discerns the modes of the straight waveguide and t the transverse components of the fields. In Eq. (30) the orthogonality relation (14) has been applied. Accordingly $\overline{P}'^{(b)}_{nn}$ is the normalized power of a quasi-guided mode.

An analog expression to Eq. (30) holds for the second transition after passing through a bent segment defined by the angle φ_0:

$$C_\mu^{(r)} \approx \frac{1}{2\overline{P}'^{(r)}_{\mu}}$$
$$\int_0^\infty \left(\sum_n C_n^{(b)} \exp\big(-j(\nu_n - j\alpha_n)\varphi_0\big)\underline{\boldsymbol{\mathcal{E}}}_{tn}\right) \times \underline{\boldsymbol{\mathcal{H}}}^{(r)}_{t\mu}\, \mathrm{d}\rho. \qquad (31)$$

Here the index (r) indicates the second straight waveguide which follows the bent waveguide.

As mentioned before, the calculation rules (30) and (31) are good approximations for small radiation loss. It is obvious that radiated fields cannot be expressed just in terms of quasi-guided modes. In this case the full mode spectrum of the bent waveguide must be regarded and an integral equation like Eq. (15) has to be solved. But often the exact solution for the radiated fields is not a matter of particular interest. The primary parameter of a transmitting channel is the outgoing power flow. If the angle φ_0 of the bent waveguide is sufficiently large, so that all radiating fields are emitted and just the quasi-guided modes contribute to the fields in the endface of the bent waveguide, then Eqs. (30) and (31) are still capable to calculate the outgoing power flow. In the following this approach is referred to as QGM-Method.

4 Numerical validation

Quasi-guided modes are exact solutions of Maxwell's equations, but they are certainly not a complete set of modes. Whether they are suited to model the power flow like proposed in Sect. 3.2 should be tested by comparison with a numerical simulation. This simulation has been done with CST MICROWAVE STUDIO® (MWS2009), i.e. the Finite Integration Technique. Because the wavelength $\lambda = 1.55\,\mu\text{m}$ is very small compared to the 90° bent waveguide trajectory, the radius R is decreased to $R = 250\,\mu\text{m}$ and $R = 125\,\mu\text{m}$ respectively, in order to reduce the memory requirements. Furthermore no straight waveguides have been considered explicitly, although the field in $\varphi = 0$ is defined by the fundamental mode of a straight waveguide. The thickness of the core is $d = 125\,\mu\text{m}$ and the refractive indices are $n_1 = 1.1$ and $n_2 = n_3 = 1.0$. For the numerical simulation a discretization with 20 lines per wavelength leads to approximately 60 millions of unknowns. Finally open boundaries have been applied to produce the results for $R = 250\,\mu\text{m}$ presented on the left side in Fig. 3.

Depicted is the magnitude of the electric field within the ϱ-φ-plane. As is it shown with $R = 250\,\mu\text{m}$ there is no radiation loss, i.e. there is almost no power flow in radial direction in $R^+ < \varrho$. And as supposed there are only minor differences in the results of the QGM-Method. The graphs on the right in Fig. 3 show the the fields in the input plane $\varphi = 0$ and in the output-plane $\varphi = \pi/2$, respectively, before and after the expansions (30) and (31), denoted by the indices $-/+$ in the keys. For the interpretation it is important to know, that 10 quasi guided modes with $k < 5$ exist in the bent waveguide, compared to 12 guided modes in the straight waveguide. The results demonstrate that the boundary conditions in $\varphi = 0$ and $\varphi = \pi/2$ are approximately fulfilled. Furthermore there is good agreement between the results of QGM-Method and the numerical simulation in $\varphi = \pi/2$.

Figure 4 shows the corresponding results for $R = 125\,\mu\text{m}$. Due to the reduced radius of the bent waveguide and since only the fundamental mode of a straight waveguide is excited

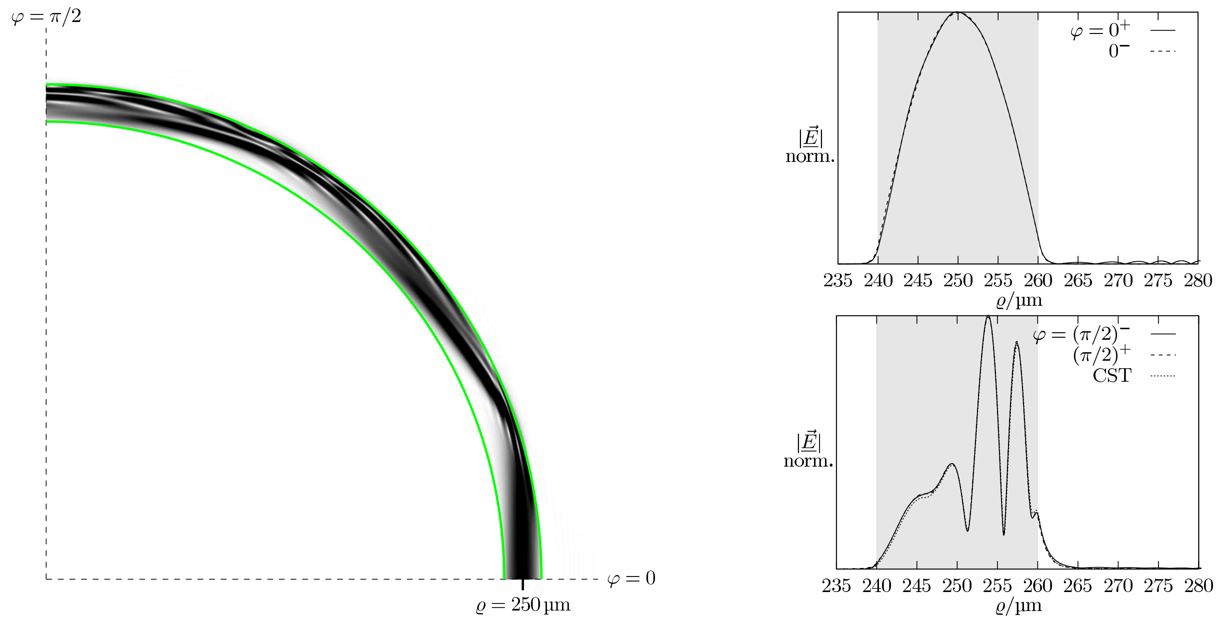


Fig. 3. Left side: Magnitude of the electric field in the ϱ-φ-plane simulated by CST MICROWAVE STUDIO®. Right side: Magnitude of the electric field in the port planes $\varphi = 0$ and $\varphi = \pi/2$ simulated by the QGM-method. The indices $-/+$ in the keys denote whether the fields are calculated before or after the expansions (30) and (31) and the key CST denotes the results of the numerical simulation. The waveguide core region is highlighted.

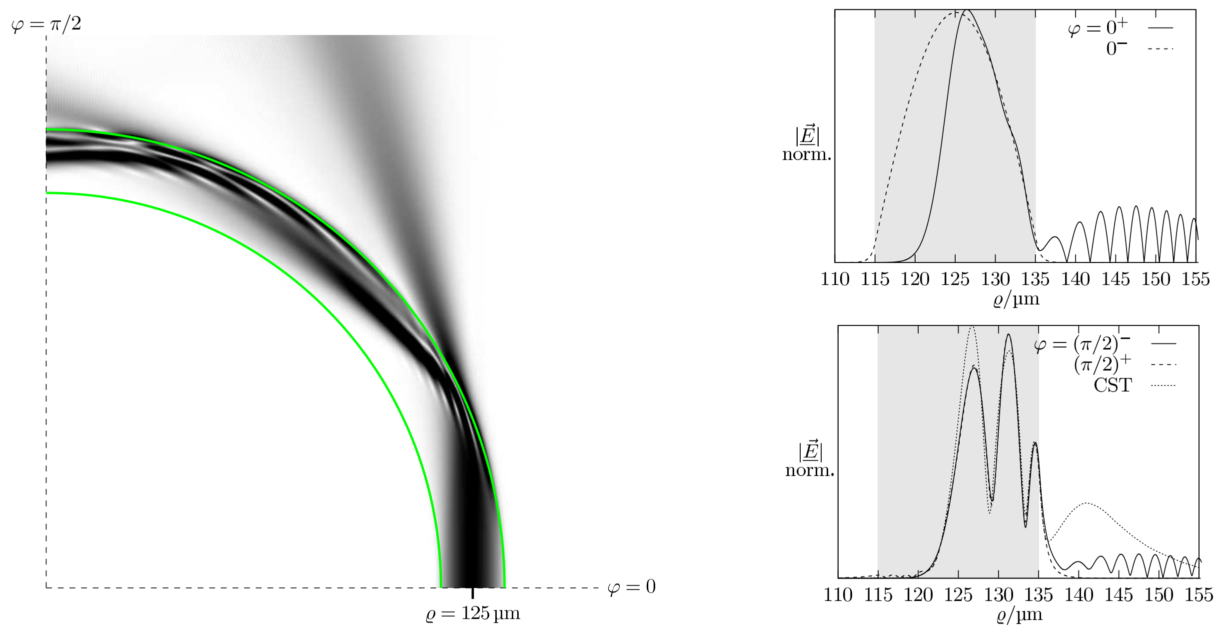


Fig. 4. See caption of Fig. 3, but $R = 125\,\mu$m.

in $\varphi = 0$, mainly at two position in the planes $\varphi \approx 30°$ and $\varphi \approx 80°$ the power radiates in form of a beam. Overall about 50% of the power coupled into the waveguide is radiated away from the core region.

As mentioned before, radiated fields cannot be expressed in terms of quasi-guided modes. Thus the QGM-Method fails especially for small angles φ and consequently the boundary conditions in $\varphi = 0$ are not satisfied. It should be noted that for $R = 125\,\mu$m only 6 quasi-guided modes with $k < 5$ remain. At the second transition, after passing the bent waveguide section of 90°, one can observe a clearly larger overlap between the different field solutions in the interface.

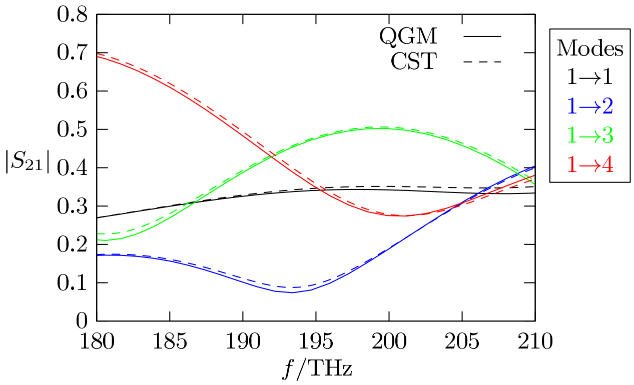


Fig. 5. Comparison of the transmission scattering parameters for the simulation with $R = 250\,\mu\text{m}$ (Fig. 3). The graphs show the coupling from the fundamental mode of the incoming waveguide to the first four modes of the outgoing waveguide.

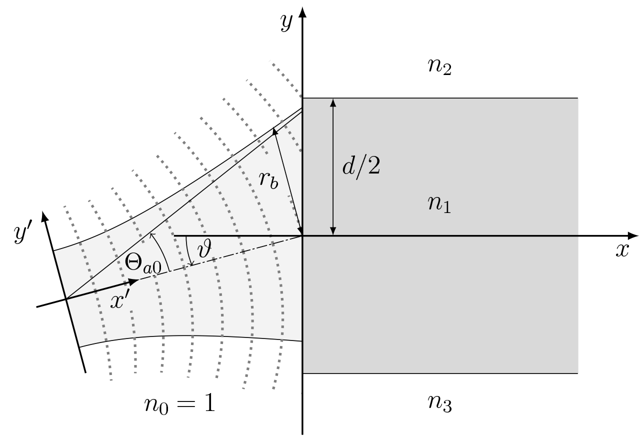


Fig. 6. Coupling conditions in the front face of the incoming straight waveguide.

That is, within the QGM-method the boundary conditions in the core region are approximately fulfilled and the overlap with the numerical solution is large. Obvious differences just remain in the outer cladding region. However, it is expected that these parts are radiated away if the wave propagates a distance within the straight waveguide, because fields in $R^+ < \varrho$ cannot significantly excite guided modes of the straight waveguide. Therefore the presented results for $R = 125\,\mu\text{m}$ are a further indication for the validity of the QGM-Method.

In addition scattering parameters have been generated from the fields in the port planes, which again show good agreement between the QGM-method and the numerical simulation. Figure 5 shows the results of the simulation with $R = 250\,\mu\text{m}$ for the first four modes of the outgoing waveguide, when in the incoming waveguide only the fundamental mode is excited. Similar agreement holds for the remaining eight modes of the straight waveguide.

5 Bending loss of multimode waveguides

The QGM-method presented in Sect. 3.2 is an efficient approach to calculate the radiation loss of a circular bended connection of two straight slab waveguides, cf. Fig. 1. In the following some exemplary simulation results for different sets of parameters will be presented. The modes of the incoming straight waveguide are excited by a Gaussian beam with a beam width $b = 2r_b$ and an asymptotic angle of divergence of $\Theta_{a0} = 5°$, see Fig. 6 (Stallein et al., 2004; Stallein, 2008).

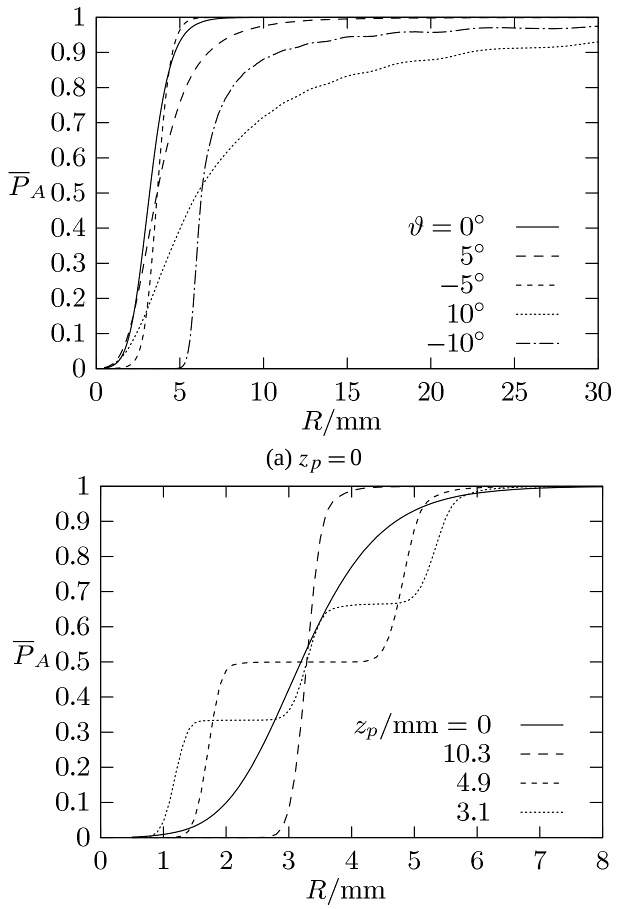
In dependency on the angle of incidence of the beam axis ϑ, different excitations of the mode spectrum result and different bending loss is expected. The bending loss $\overline{P}_A$ is defined as the ratio of the power in the guided modes of the outgoing and the incoming straight waveguide, thus the length of the outgoing waveguide is arbitrary. For the results in Fig. 5 the length of the incoming waveguide z_p tends to zero and in Fig. 5 some special values z_p have been used. Further parameters are the ratio of core thickness and beam width $d/b = 1.5$, the wavelength $\lambda = 850\,\text{nm}$, and the refractive indices $n_1 = 1.57$ and $n_2 = n_3 = 1.55$.

Figure 7 shows results for the bending loss $\overline{P}_A$ in dependency on the radius R for a 90° bend. In Fig. 5 various angles of incidence of the Gaussian beam ϑ have been considered, while the core thickness is constant $d = 75\,\mu\text{m}$. It becomes clear that the bending loss usually increases if the coupling conditions are not ideal, i.e. $|\vartheta| > 0°$. An exception is the case of a small rotation of the beam axis against the direction of the bend, which is the negative φ-direction.

Finally Fig. 5 shows some effects of multimode interference. The core thickness is once again $d = 75\,\mu\text{m}$. Because of the symmetrical stimulation with $\vartheta = 0°$, the field pattern within the incoming straight waveguide has some distinctive local maxima (Soldano and Pennings, 1995). If the length z_p is chosen such that one, two, or three maxima exist at the end of the incoming waveguide, then the dependency of the bending loss on the radius R is in the form of steps.

6 Conclusions

The theory of quasi-guided modes (QGM) is an important element in the modeling of dielectric bent waveguides. Compared with the well know leaky mode ansatz it is advantageous, because the eigenvalue equation is real valued. Nevertheless the deviations are small, e.g. all results of Sect. 5 can also be generated by means of leaky modes. The analytic calculation with a limited and discrete set of modes is time-efficient in contrast to a numerical simulation, because the optical wavelength is very small compared with the length of the waveguide trajectory. For the purpose of validation a comparison with the results of CST MICROWAVE



(a) $z_p = 0$

(b) $\vartheta = 0°$

Fig. 7. Normalized output power after passing through a 90° bend: **(a)** for different angles of incidence ϑ; **(b)** for different lengths z_p of the incoming waveguide.

STUDIO® (MWS2009) confirmed the basic suitability of the QGM-method. For large radii of curvature, i.e. small bending loss, the QGM-method is quite a good approximation for the fields in the bent waveguide. And even though the exact solution of a radiation problem of course requires the consideration of the complete continuous mode spectrum, it is possible to calculate the field at the end of the bent section, if the angle of the bend is sufficiently large. In this case all radiating parts of the fields have already passed the core region and only the quasi-guided modes contribute to the fields in the output port.

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