Combined lumped element network and transmission line synthesis for passive microwave structures

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Abstract. Compact circuit models of electromagnetic structures are a valuable tool for embedding distributed circuits into complex circuits and systems. However, electromagnetic structures with large internal propagation delay are described by impedance functions with a large number of frequency poles in a given frequency interval and therefore yielding equivalent circuit models with a high number of lumped circuit elements. The number of circuit elements can be reduced considerably if in addition to capacitors, inductors, resistors and ideal transformers also delay lines are included. In this contribution a systematic procedure for the generation of combined lumped element/delay line equivalent circuit models on the basis of numerical data is described. The numerical data are obtained by numerical full-wave modeling of the electromagnetic structure. The simulation results are decomposed into two parts representing a lumped elements model and a delay line model. The extraction of the model parameters is performed by application of the system identification procedure to the scattering transfer function. Examples for the modeling of electromagnetic structures are presented.

1 Introduction

The importance for applying circuit-theoretic multi-port concepts in the modeling of wireless communication links already has been pointed out in Ivrlac and Nossek (2010). Different from simple information theoretic models multi-port models allow to consider the energy flow and the coupling of antenna elements. The use two-port models of antennas already has been discussed in literature (Rogers et al., 2003; Boryssenko and Schaubert, 2007; Aberle, 2008). These models yielded an improved description of antennas, however,

due to the limited number of lumped elements the antenna could be modeled in a narrow frequency band only.

Distributed circuits can be modeled also in a broad frequency band with arbitrary accuracy using lumped element network models. A general way to establish network models is based on modal analysis and similar techniques (Russer, 2006; Felsen et al., 2009; Russer et al., 2010). In Russer et al. (2010) also the modeling of lossy structures on the basis of Brune's equivalent circuit realization has been discussed. However, electromagnetic structures with large internal propagation delay are described by impedance functions with a large number of frequency poles in a given frequency interval and therefore yielding equivalent circuit models with a high number of lumped circuit elements. The number of circuit elements can be reduced considerably if in addition to capacitors, inductors, resistors and ideal transformers also delay lines are included. A combined lumped elements and delay line approach already has been presented in Shevgunov et al. (2008).

In this contribution a systematic procedure for the generation of a combined lumped element/delay line equivalent circuit model for a wireless transmission link on the basis of numerical data is described. The numerical data are obtained by numerical full-wave modeling of the electromagnetic structure. The simulation results are decomposed into two parts representing a lumped elements model and a delay line model. The extraction of the model parameters is performed by application of the system identification procedure to the scattering transfer function. Examples for the modeling of electromagnetic structures are presented.

2 Model of a wireless link

Let us consider the easiest case of a wireless transmission link consisting of two antennas connected in the far-field over free-space. Figure 1 shows this arrangement schematically. In $\mathbf{Z}$ representation the voltage-current relations are given by
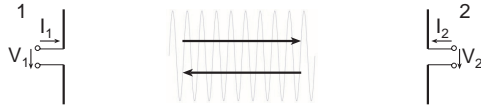
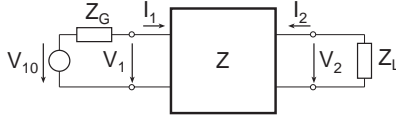**Fig. 1.** Wireless transmission link with two antennas.

Fig. 2. Two-port representation of the wireless transmission link with source and load.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}. \tag{1}$$

Figure 2 shows the block diagram of the wireless link for the case where antenna 1 is the transmitting antenna and antenna 2 is the receiving antenna. The transmitter has the open source voltage V_{10} and the source impedance Z_G. The receiving antenna is terminated with the load impedance Z_L. Under these conditions the input and output currents I_1 and I_2, respectively are given by

$$I_1 = \frac{Z_{22} + Z_L}{(Z_{11} + Z_G)(Z_{22} + Z_L) - Z_{12}Z_{21}} V_{10}, \tag{2}$$

$$I_2 = \frac{Z_{21}}{(Z_{11} + Z_G)(Z_{22} + Z_L) - Z_{12}Z_{21}} V_{10}. \tag{3}$$

Figure 3 shows the schematic two-port representation of the link with source and load where Z_{12} and Z_{21} are represented by current controlled voltage sources. The input impedance Z_{in} of the wireless link two-port $\mathbf{Z}$ terminated by Z_L is given by

$$Z_{in} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22} + Z_L}. \tag{4}$$

The available power P_a of the transmitter is given by

$$P_a = \frac{|V_{10}|^2}{8\Re\{Z_G\}}. \tag{5}$$

The power delivered to the load R_L is

$$P_L = \frac{1}{2}\Re\{Z_L\}|I_2|^2. \tag{6}$$

The transducer power gain is defined as

$$G_t = \frac{P_L}{P_a} = \frac{4|Z_{21}|^2\Re\{Z_G\}\Re\{Z_L\}}{|(Z_{11} + Z_G)(Z_{22} + Z_L) - Z_{12}Z_{21}|^2}. \tag{7}$$

In a wireless transmission link the reaction of the receiving antenna on the transmitting antenna usually is negligible, i.e. $|(Z_{11} + Z_G)Z_{22}| \gg |Z_{12}Z_{21}|$. In this case we can approximate the transducer power gain by the unilateral power gain.
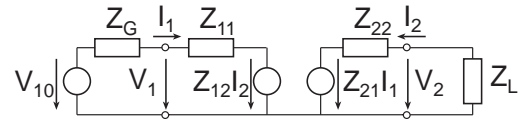

Fig. 3. Two-port representation of the wireless transmission link with source and load where Z_{12} and Z_{21} are represented by current controlled voltage sources.
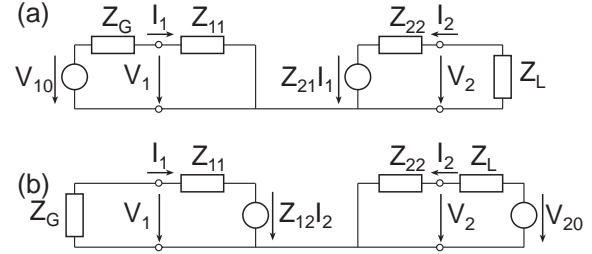

Fig. 4. Two-port representation of the unilateralized wireless transmission link with source and load **(a)** with antenna 1 as the transmitting antenna, **(b)** with antenna 2 as the transmitting antenna.

Depending on the direction of transmission we can replace the reciprocal wireless link models by the non-reciprocal models shown in Fig. 4. In this case the power gain G_t is approximated by the unilateral power gain

$$G_{tu} = \frac{4|Z_{21}|^2\Re\{Z_G\}\Re\{Z_L\}}{|(Z_{11} + Z_G)(Z_{22} + Z_L)|^2}. \tag{8}$$

We note that in this approximation the antenna feed impedances of antenna 1 and antenna 2 are given by Z_{11} and Z_{22} respectively.

The unilateral power gain exhibits its maximum value $G_{tu,max}$ for power matching at the input and output, i.e. $Z_G = Z_{11}^*$, $Z_L = Z_{22}^*$ and is given by

$$G_{tu,max} = \frac{|Z_{21}|^2}{4\Re\{Z_{11}\}\Re\{Z_{22}\}}. \tag{9}$$

3 The two-port antenna model

As long as the coupling of the antenna with the other antenna is neglected the antenna can be considered as a passive one-port. If the antenna is coupled with one or more other antennas over the far-field only, the driving point impedance Z_{in} of this antenna one-port will not be influenced by the other antennas. In the following we assume the antenna itself to be lossless. The real part of the antenna driving point impedance in that case is only due to the active power radiated by the antenna. Z_{in} is a positive real impedance function. A one-port with a positive real impedance function can be represented by a reactive reciprocal two-port $\mathbf{Z_R}$ terminated by a real resistor R_r (Guillemin, 1957). Figure 5 shows the block
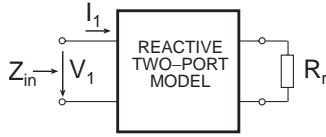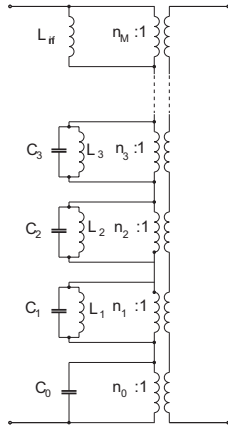
Fig. 5. Two-port antenna model.

Fig. 6. Foster realization of the reactive two-port.

Fig. 7. Photograph of the open slot antenna (Yordanov and Russer, 2010).
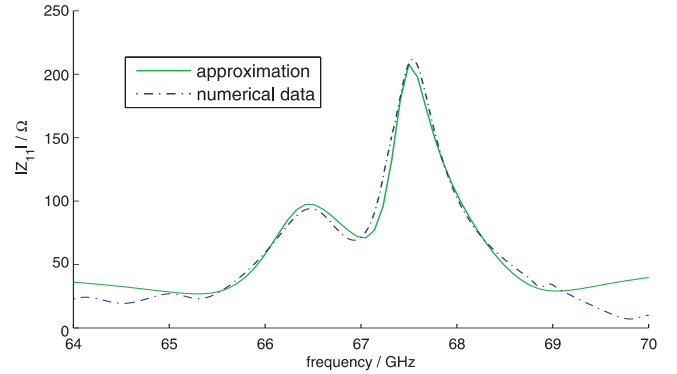
Fig. 8. $|Z_{11}|$ of the wireless transmission link.

diagram of the two-port antenna model. The determination of the positive rational impedance functions of the two-port was performed by system identification methods (Kuznetsov et al., 2004; Russer et al., 2010).

An approximation of the lossless reciprocal two-port based on a finite number of poles of its two-port reactance matrix Z_R can be realized with a finite number of capacitors, inductors and ideal transformers. Figure 6 shows the canonical Foster realization of the reactive two-port (Russer, 2006; Felsen et al., 2009; Russer et al., 2010).

The complete impedance matrix describing a circuit with a finite number of poles is obtained by parallel connecting the circuits describing the individual poles. In the canonical Foster representation, the impedance matrix $Z(\omega)$ is given by

$$Z_\lambda(\omega) = \frac{1}{j\omega C_0} B_0 + \sum_{\lambda=1}^{N} \frac{1}{j\omega C_\lambda} \frac{\omega^2}{\omega^2 - \omega_\lambda^2} B_\lambda + j\omega L_\infty B_\infty.$$

(10)

The frequency independent rank 1 matrices B_λ are given by

$$B_\lambda = \begin{bmatrix} n_\lambda^2 & n_\lambda \\ n_\lambda & 1 \end{bmatrix},$$

(11)

where the n_λ are the turns ratios of the transformers.

We have applied the methods described in this paper to model communication links between monolithic integrated on-chip antennas. Figure 7 shows a photograph of an open-chip antenna with CMOS circuits under the antenna electrode (Yordanov and Russer, 2010). The antennas have been measured on-wafer and diced.
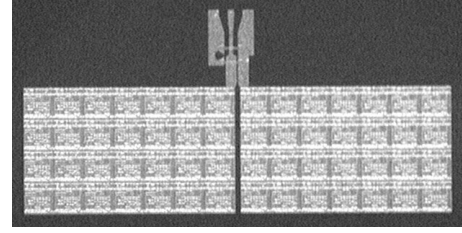
Figure 8 shows the comparison of the $|Z_{11}|$ data obtained from numerical simulation with the data computed from the two-port model according to Fig. 5 with the reactive Foster two-port model shown in Fig. 6. The numerical full-wave simulations have been performed using CST software. Based on four pairs of poles and two single poles at zero and infinity the two-port model provides an accurate model of Z_{11} for the frequency band from 65 GHz to 69 GHz. The frequency range may be extended by increasing the number of poles.

4 The wireless link model

To extend the two-port antenna model to the full wireless link model we start from the assumption that the losses in the antenna are mainly radiation losses. In this case we can assume that the reactive two-port in Fig. 6 models the near-field of the antenna whereas the real resistor R_r models the energy dissipation in the far-field. Considering dispersion-free single-path propagation between the two antennas, we can model the free-space propagation of the electromagnetic wave by a dispersion-free lossy transmission line (TL). The length of this transmission line corresponds to the distance between the antennas and the attenuation follows from Friis transmission formula (Russer, 2006).

This yields the model of the complete wireless transmission link shown in Fig. 9. The scattering matrix of the lossy
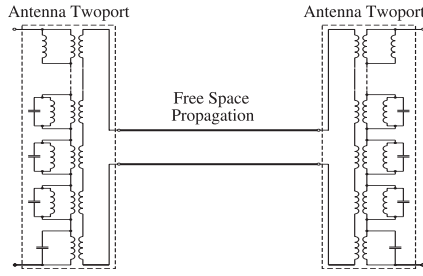
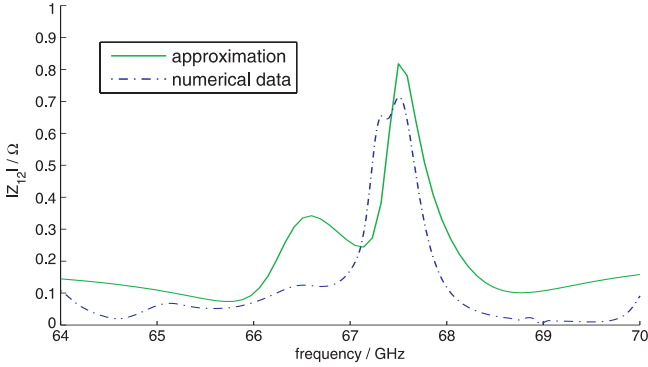**Fig. 9.** Model of the complete wireless transmission link.

Fig. 10. $|Z_{12}|$ of the wireless transmission link.

transmission line with wave impedance R_r is given by

$$S_L = a \frac{\exp(-jkr)}{kr} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \tag{12}$$

where a represents the attenuation according to the Friis formula and r is the distance between the antennas. The corresponding **Z**-matrix is given by

$$\mathbf{Z}_L = Z_0 \begin{bmatrix} \frac{1+\frac{a^2\exp(-2jkr)}{(kr)^2}}{1-\frac{a^2\exp(-2jkr)}{(kr)^2}} & \frac{\frac{a\exp(-jkr)}{kr}}{1-\frac{a^2\exp(-2jkr)}{(kr)^2}} \\ \frac{\frac{a\exp(-jkr)}{kr}}{1-\frac{a^2\exp(-2jkr)}{(kr)^2}} & \frac{1+\frac{a^2\exp(-2jkr)}{(kr)^2}}{1-\frac{a^2\exp(-2jkr)}{(kr)^2}} \end{bmatrix}. \tag{13}$$

For $\frac{a\exp(-jkr)}{kr} \ll 1$ this can be approximated by

$$\mathbf{Z}_L \approx Z_0 \begin{bmatrix} 1 & \frac{a\exp(-jkr)}{kr} \\ \frac{a\exp(-jkr)}{kr} & 1 \end{bmatrix}. \tag{14}$$

Figure 10 shows the comparison of the numerical data of $|Z_{12}|$ obtained from the full-wave simulation of the wireless transmission link with the data computed from the model shown in Fig. 9. The stronger deviation between these data, compared with the case of $|Z_{11}|$ may be due to losses in the antenna and will be subject to further investigations.

5 Conclusions

We have presented an accurate method for generation of compact circuit models of wireless transmission links, where the antenna structure and the near-field regions are modeled by reactive lumped element networks and the far-field propagation by dispersion-free lossy transmission lines. Further investigations will extend this work towards multi-antenna systems and multi-path propagation. This should provide a framework for physical modeling of MIMO systems.

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